Presumption of patent validity and litigation incentives*

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Research highlights

- We model how the presumption of patent validity biases judicial priors and influences litigation strategies and outcomes.
- We find that the presumption can either encourage or deter litigation, depending on underlying patent merit and legal costs.
- The presumption reduces resource dissipation but may increase judicial errors when there is high uncertainty regarding validity.
- These findings support limiting the presumption's use, especially in complex sectors where invalid patents are more frequently granted.

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Abstract

We analyze the effects of the presumption of patent validity on litigation incentives and outcomes. We develop a litigation game between a patent holder and an alleged infringing firm. A court resolves the dispute if there is a trial. We model the court's decision-making as a learning process based on evidence and consider the presumption as a factor influencing the court's prior belief of patent validity. The presumption affects the trial outcome in two ways—directly by biasing the prior, and indirectly by affecting the incentives to invest in evidence-seeking activities. We show that its effect on the likelihood of trial is ambiguous. Moreover, when patent validity is uncertain, the presumption generates a trade-off: it reduces resource dissipation, but increases the probability of judicial errors. With pre-trial settlement, even low-merit patents are profitably asserted, and a stronger presumption raises the settlement payments extracted through credible enforcement threats. Taken together, our results suggest a cautious or limited application of the presumption, especially in environments where patent validity is highly uncertain.

JEL classification: C72, D74, D83, K11, K41, O34

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1 Introduction

Granted patents generally enjoy a presumption of validity in court proceedings (Seaman, 2019). The burden of proof falls upon the infringer, who has to provide relevant facts about patent invalidity and convince the court with clear and convincing evidence, which is a higher standard of proof than the "preponderance of evidence" typically required in civil lawsuits.[1] In common law countries, by statute "a patent shall be presumed valid" and the "burden of establishing invalidity of a patent or any claim thereof shall rest on the party asserting such invalidity" (35 USC ∮282, 1952).[2] A similar presumption holds for European patents and Supplementary Protection Certificates (Graham et al., 2002; Seaman, 2019).

There are different theoretical justifications in support of the presumption of [patent] validity—the most common one being deference to the patent office's expertise in evaluating patent applications, along with its agency flexibility and political accountability

[1]To clarify some of the terminology that will be used in our analysis, the standard of proof is the level of certainty and the amount of evidence necessary to prove a claim in a trial; the burden of proof identifies the party who must offer evidence to raise a claim in litigation (Sanchirico, 1997). These concepts are practically interdependent but theoretically distinct (Clermont and Sherwin, 2002; Schwartz and Seaman, 2013; Guerra et al., 2019b; 2022).

[2]The Congress amended Section 282 in 1965, 1975, 1995, and 2011. The sentences: "A patent shall be presumed valid," and "The burden of establishing invalidity of a patent or any claim thereof shall rest on the party asserting such invalidity," remained in the statute. See, e.g., *Microsoft Corp. v. i4i Limited Partnership*, 131 S.Ct. 2238 (2011).

(Devlin, 2008; Seaman, 2019).[3] Notwithstanding this general rationale, the application of the presumption is neither uniform across jurisdictions nor across legal proceedings within a jurisdiction (Ottoz, 2019). Differences across national jurisdictions are substantial, especially within Europe (Graham et al., 2002; Graham and Van Zeebroeck, 2013; Cremers et al. 2017). For example, the Netherlands apply no presumption, whereas Sweden and Denmark generally apply a strong presumption of validity unless there is clear evidence of invalidity. Recent case laws have also considered a reversed presumption, namely a presumption of invalidity (e.g., *Syral Belgium v. Roquette Frères).*[4]

Differences across legal proceedings within a jurisdiction are substantial as well. In the U.S., the presumption of validity is applied in court litigation, whereas no presumption or a "weakened" presumption is applied in post-issuance administrative proceedings before the U.S. Patent and Trademark Office's (USPTO) Patent Trial and Appeal Board (PTAB). These proceedings—through which the public may ask the patent office to reassess the validity of granted patents—include inter partes review, post-grant review, and ex parte re-examination (Rai and Vishnubhakat, 2019; Helmers and Love, 2023).[5] For example, in the inter partes review—which has been introduced by the America Invents Act on September 16, 2012—a petitioner can challenge the validity of a U.S. patent under a "preponderance of evidence" standard, showing that claims are more likely unpatentable than not (Helmers and Love, 2023).[6] This significantly reduces the standard of proof, compared to patent litigation in court where the "clear and convincing evidence" standard is applied. Consequently, an infringement defendant may prefer to challenge patent validity in post-issuance administrative proceedings, rather than in or in parallel with litigation in district courts (Rai and Vishnubhakat, 2019; Seaman, 2019; Helmers and Love, 2023).

Beyond the discrepancies in the application of the presumption of validity across jurisdictions and legal proceedings, several critiques have been raised upon the mere presence of the presumption. Some scholars argued that the presumption should be abolished because it is not simply a procedural device, but rather a powerful mechanism

[3]See also *Cuozzo Speed Techs. v. Lee*, 136 S. Ct. 2131, 2144 (2016).

[4]*Syral Belgium v. Roquette Frères* (Supreme Court, Belgium, 12 September 2014, Case No. C.13.0232) challenged the *prima facie* validity of a patent based on a decision from other European jurisdictions (UK and France) invalidating the patent. In this case, a foreign legal decision may generate a presumption of invalidity for the Belgian part of the patent.

[5]Regarding inter partes review, "In an inter partes review [. . .], the petitioner shall have the burden of proving a proposition of unpatentability by a preponderance of the evidence." 35 U.S.C. ∮ 316(e). See also *In re* Global Tel*link Corp., IPR 2014-00493, 2014 WL 4715524, Sept. 17, 2014 ("There is no presumption of validity as to the challenged claims in an inter partes review."). Regarding ex parte re-examination, "the standard of proof – a preponderance of evidence – is substantially lower than in a civil case [and] there is no presumption of validity." (*In re* Swanson, 540 F.3d 1368, 1377, Fed. Cir. 2008).

[6]35 U.S.C. § 316(e) (inter partes review), 326(e) (post-grant review). See also the Trial Practice and Procedure Rules, confirming that "the default evidentiary standard is a preponderance of the evidence" (37 CFR § 42.1(d)). For more details about the inter partes review process, see, e.g., Helmers and Love (2023).

for injecting pro-patentee bias (Bohrer, 2004; Bock, 2014). An explicit statement in jury instructions that a patent is presumed valid makes juries less likely to invalidate patents (Moore, 2002). This is not problematic if the patent was correctly granted: in this case, the presumption protects the patent holder from any frivolous (or, non-meritorious) infringement litigation (Seaman, 2019). The problem arises when patent offices make evaluation errors and grant patents that, on their merits, should not have been issued in the first place (de Rassenfosse et al., 2021).[7] These patents are referred to in the literature as "latently invalid" or "incorrectly granted" patents (Henkel and Zischka, 2019), and also as "bad" or "weak" patents (Choi and Gerlach, 2015; Lei and Wright, 2017).

The problem of latently invalid patents is sizable, especially in the U.S. (Commission, 2004; Lemley and Shapiro, 2005; The Economist, 2015).[8] Judicial review is generally one solution to correct latently invalid patents. Yet, only a small fraction of all patents is litigated (roughly 1.5%, while only about 0.1% proceed to trial), as emphasized by Lemley and Shapiro (2005). This selection into litigation and trial reflects both the high cost of district court litigation and institutional features such as the presumption of validity (American Intellectual Property Law Association, 2013; Helmers and Love, 2023).

Rather than entering litigation, innovators may find it more convenient to pay licensing and transaction costs of bargaining to reach a private agreement with the patentee, thereby leaving the latently invalid patent in the market (Kesan and Gallo, 2006). Related theoretical work formalizes the strategic implications of patent-validity uncertainty for licensing. Encaoua and Lefouili (2009) model two-part-tariff licensing of uncertain patents in an oligopoly, showing that weak patents can be over- or undercompensated depending on the royalty rate that deters litigation. Amir et al. (2014) show that holders of weak patents prefer per-unit royalties, which raise aggregate profits by relaxing downstream competition. These contributions complement the probabilistic-patents perspective of Lemley and Shapiro (2005) by clarifying how validity uncertainty shapes the choice and terms of licensing contracts. More broadly, patent-validity uncertainty compounds the uncertainties of the innovation process by exposing innovators to the risk

[7] Patent evaluation errors by patent offices as the USPTO may occur for different reasons, including low accuracy levels of reviewing patent applications, or inefficient incentives to grant valid patents (e.g., absence of penalties for incorrectly issued patents). Evaluation errors are particularly common in high technology sectors, which are not areas of traditional patenting, and therefore it is more difficult for the patent office to gather prior art information (Kesan and Gallo, 2006). Consistent with this, Metro et al. (2024) show that prosecution and examination characteristics, including the number of office actions and examiner stringency, are significant predictors of validity and infringement outcomes in litigation. On the risk of errors by both patent examiners at patent offices and judges at civil court, see Buzzacchi and Scellato (2008).

[8] The problem is also present in Europe, but less pronounced (Cremers et al., 2017; Henkel and Zischka, 2019). For example, Cremers et al. (2017) reported that about 30% of appealed patent suits have their initial decision overturned; Henkel and Zischka (2019) found a 75% invalidity rate of appeals at the German Federal Patent Court between 2000 and 2012. See also Palangkaraya et al. (2011), which analyzed patent applications granted by the USPTO and examined at both the European Patent Office and Japanese Patent Office during the 1990s. Their estimates reveal that 9.8% of patents were incorrectly granted.

of non-meritorious litigation (Farrell and Shapiro, 2008; de Rassenfosse et al., 2021). A major driver of non-meritorious patent infringement litigation is represented by non-practicing entities (NPE; also called "patent trolls")—that is, patent holders whose sole purpose is to enforce patent rights to threaten litigation and demand licensing fees, rather than producing or selling products or services (Pénin, 2012; Ganglmair et al., 2022).[9] The presumption of validity may have the double-edged effect of leveraging NPEs' litigation tactics and their abilities to extract licensing payments from producing firms (Patent Quality Improvement Hearings, 2003, p. 4).

The debate upon the presumption has not been limited to scholarly contributions. During court litigation cases, judges and juries discussed the application of the presumption and the level of validity which should be applied to issued patents (for a review, see Klimczak, 2012). For example, the Supreme Court articulated and confirmed the application of the presumption at common law in several legal cases—one of the most cited being *Microsoft Corp. v. i4i Ltd. Partnership.*[10] In other legal cases, the Supreme Court stated that the presumption can be weakened or even eliminated when the patent office did not or was not able to consider relevant prior art in its review of patent applications.[11]

The legal debates on the presumption, along with the discrepancies in its application across jurisdictions and legal proceedings, circle back to a basic, yet fundamental question: *What are the effects of the presumption on litigation incentives?* To our knowledge, no prior studies thus far formally addressed this question. This gap in the literature is problematic, especially considering that legal and empirical studies have suggested the important role of legal presumptions in patent litigation (Chatlynne, 2010; Schwartz and Seaman, 2013; Bock, 2014; Henkel and Zischka, 2019; Seaman, 2019),[12] and some scholars and governmental institutions have repeatedly advocated reforms to overcome the presumption in court proceedings (Devlin, 2008; Ottoz, 2019; Seaman, 2019). A particularly influential contribution is Lichtman and Lemley (2007), who argue that the burden of proving validity should fall on the patent holder, especially when litigation constitutes the first serious examination of the patent's merit. Their argument highlights

[9] A classic example of a legal case involving a patent troll is *NTP, inc. v. Research in Motion*, 397 F. Supp. 2d 785 (E.D. Va. 2005), where a NTP (a Virginia-based patent holding company) sued Research in Motion (RIM, the manufacturers of Blackberry) for patent infringement over its Blackberry devices. Even if the patent office found that three disputed patents should have not been granted in the first place, the NTP won an out-of-court settlement of US$ 612 million from RIM. For a discussion of this and other patent litigation cases involving patent trolls, see, e.g., Chan and Fawcett (2005).

[10] *Microsoft Corp. v. i4i Ltd. Partnership*, 131 S. Ct. 2238, 564 U.S. 91, 180 L. Ed. 2d 131 (2011). For an extensive discussion on this legal case, see Schwartz and Seaman (2013). See also *Radio Corp. of America v. Radio Engineering Laboratories, Inc.*, 293 U.S. 1, 55 S. Ct. 928, 79 L. Ed. 163 (1934); *Austin Machinery Co. v. Buckeye Traction Ditcher Co.*, 13 F.2d 697 (6th Cir. 1926); *Coffin v. Ogden*, 85 U.S. 120, 21 L. Ed. 821 (1874); *Sciele Pharma Inc. v. Lupin Ltd.*, 684 F.3d 1253 (Fed. Cir. 2012).

[11] See, e.g., *Manufacturing Research Corp. v. Graybar Elec. Co.*, 679 F.2d 1355 (11th Cir. 1982); *Heyl & Patterson, Incorporated v. McDowell Company*, 317 F.2d 719 (4th Cir. 1963).

[12] Some theoretical contributions analyzed the effects of legal presumptions on individuals' choices in other contexts (e.g., in tort settings; Bernardo et al., 2000; Demougin and Fluet, 2008; Guerra et al., 2022).

a central policy tension: removing or weakening the presumption may improve litigation efficiency by reducing nuisance suits and facilitating the invalidation of weak patents, but it may also undermine the reliability of granted patent rights, which firms rely on for commercialization and investment decisions. Understanding this tension requires a formal account of how the presumption shapes litigation incentives and welfare—a gap that motivates our analysis.

In this article, we seek to fill this knowledge gap by providing an analytical framework that explains how the legal presumption affects litigation incentives and its efficiency implications. We present a tractable patent-litigation game between a patent holder and a potential infringing firm. A non-strategic decision-maker, for example, a judge or a jury in case of court proceedings, resolves the dispute if there is a trial. The game consists of three sequential stages: the patent holder's decision to sue; the infringing firm's decision to defend; the trial stage where parties invest resources to gather and present evidence in order to persuade the court to make a favorable decision. The court's prior belief of patent validity is influenced by the presumption criterion, whereas the posterior belief takes the evidence produced during the litigation trial into account. The presumption affects the outcome of a trial in two ways—by biasing the prior, and by affecting the incentive to invest in evidence-seeking activities.

We characterize the competing parties' litigation decisions and their investment decisions that arise in equilibrium. We show that the effect of the presumption on the possibility of a litigation trial can be ambiguous. We further analyze its effect on two important features of a litigation trial—resource dissipation and error of judgment. Both resource dissipation and error of judgment are inefficient for the society. One of the key insights from our analysis is that the presumption has countervailing efficiency effects when there is high uncertainty about the patent's objective merit, e.g., in contexts where the examination of patent applications is complex and invalid patents are granted more frequently by the patent office (e.g., in high technology sectors). The countervailing effects arise because the presumption biases the evidence-production contest in favor of the patent holder: this dampens competition and thereby reduces resource dissipation, but it also blunts incentives to gather new evidence whose social learning value is highest when merit is uncertain.

Two stylized mappings help connect these forces to observable litigation environments. First, consider a *high-uncertainty* setting, such as software or business-method patents, where claim boundaries are often ambiguous, prior art is diffuse and difficult to identify, and patent offices face well-documented challenges in evaluating applications (Kesan and Gallo, 2006). In such environments, objective merit is difficult to assess ex ante, while marginal information generated during litigation is highly valuable and outcomes are particularly sensitive to evidence and procedural rules. Our model implies that, in these settings, a stronger presumption can reduce litigation costs but may also weaken incentives to produce information precisely when its social value is greatest, reflecting the countervailing effects highlighted in our analysis. In contrast, consider a *lower-uncertainty* setting, such as pharmaceutical composition-of-matter patents, where claim scope is typically narrow, prior art is well cataloged, and validity is comparatively easy to evaluate. In these environments, the marginal social value of additional trial evidence is lower, so a presumption-induced reduction in evidence production entails a smaller loss in accuracy while still economizing on dispute costs.

Our baseline model does not include a settlement stage. This is a deliberate choice: under complete information, settlement eliminates trials on the equilibrium path, as parties can always negotiate a transfer that both prefer to the costly trial outcome (Spier, 2007). Analyzing the trial subgame directly allows the baseline to isolate the presumption's effects on evidence-seeking incentives, resource dissipation, and judicial error. We then extend the model to add a pre-trial bargaining stage. In this extension, disputes that reach bargaining are resolved through agreement in equilibrium, and the trial-stage objects—the expected probability of prevailing and equilibrium evidence expenditures—serve as threat-point primitives governing settlement transfers and the plaintiff's litigation leverage.

This reinterpretation has direct policy relevance: once settlement is available, the no-litigation and litigation-trial regimes of the baseline are absorbed by settlement—the patent holder always sues and the defendant always defends, so disputes are resolved through negotiated transfers rather than trials. The default-judgment regime survives only as an indifference case when the settlement payment equals full damages. Consequently, the policy-relevant margin shifts from *whether* disputes arise to *how large* the settlement transfer is, and a stronger presumption raises the equilibrium transfer for all priors at which the damages cap does not bind. The mechanism aligns with the empirical regularity that most patent disputes settle rather than proceed to trial, and it speaks to concerns about non-meritorious litigation threats, particularly those associated with NPE activity. Empirical work documents substantial private and social costs of assertion-focused NPE behavior (Bessen et al., 2011; Ganglmair et al., 2022). Within our transfer-focused framework, a monetization-oriented NPE is a natural application: the presumption strengthens the plaintiff's bargaining position by improving expected trial outcomes and altering evidence-seeking incentives, thereby shifting equilibrium settlement transfers. Other NPE types, such as university technology-transfer offices pursuing commercialization objectives, or privateering entities introducing principal–agent dynamics, fit less directly and lie beyond our current scope.

The baseline model abstracts from any direct impact of litigation on market structure. This assumption is adopted for tractability and is best interpreted as capturing environments in which the primary private stakes can be approximated as monetary transfers, rather than settings where adjudication (or preliminary relief) directly shapes market outcomes through exclusionary remedies. In Section 5.2, we introduce a reduced-form mapping that allows adjudication errors and enforcement threats to generate downstream market effects, such as delayed adoption or exclusion, without embedding a full product-market structure.

Our model of the litigation game is inspired by Skaperdas and Vaidya (2012): we model the litigation-trial stage game as a contest with Bayesian learning, a probabilistic decision rule, and a stochastic evidence-production function.[13] However, our approach departs from theirs in several ways. First, we introduce the presumption as a factor that influences the decision-maker's prior belief of patent validity, before examining any evidence. This approach helps to study how the tension between adjusting belief due to the presumption and that due to learning from evidence, affects the litigation incentives. Second, we consider a specific stochastic evidence-production function that gives closed-form solutions of the equilibrium strategies, which enable us to study the comparative static effects in a tractable manner. Last but not least, our focus is different from theirs. Skaperdas and Vaidya (2012) intend to provide an information-based foundation of the additive contest success function (CSF) in a litigation contest, whereas we consider the Bayesian inferential process at the trial stage as a building block, and focus on how a biased belief arising from a legal presumption affects litigation incentives.[14]

In much of the existing works involving contests with fixed prizes, efforts are non-productive and purely rent-seeking in nature (Guerra et al., 2019a). In our framework, contest efforts can lead to hard information that fosters a proper allocation of property rights. In this sense, we contribute to the informational lobbying literature (e.g., Austen-Smith and Wright, 1992; Bennedsen and Feldmann, 2002; 2006; Lagerlöf, 2007), with a distinct focus on how a biased prior belief affects information-seeking and litigation incentives. Our approach also differs from Bayesian persuasion in contests (Zhang and Zhou, 2016; Clark and Kundu, 2021), where a designer typically commits to an information-disclosure mechanism; in our model, parties themselves produce hard information to persuade the decision-maker. Heterogeneity among players is commonly acknowledged as a limiting factor to investments in contest (Chowdhury et al., 2023). Our model share common features with this literature, particularly in analyzing how the inherent asymmetry due to the presumption bias extends to litigation incentives.

The rest of the article is organized as follows. Section 2 introduces the model setup

[13]On modeling contest success functions in litigation contests, see, among many others, Hirshleifer and Osborne (2001); Farmer and Pecorino (1999).

[14]The literature on contests, particularly the Tullock CSF is extensive and has been applied to various economic, legal, and political contexts (e.g., Plott, 1987; Chowdhury and Sheremeta, 2011a,b; Skaperdas and Vaidya, 2012; Chen and Rodrigues-Neto, 2023). Our contribution lies in the specific application of the contest framework to legal presumptions in patent litigation. For exhaustive reviews of Tullock's works and the contest literature, see, e.g., Parisi et al. (2017); Guerra et al. (2019a, 2022), Guerra et al. (2023). Our analysis falls within this well-established framework, emphasizing the unique role of legal presumptions in shaping litigation incentives.

and assumptions. Section 3 characterizes the equilibrium. Section 4 examines the effects of the presumption on litigation incentives, resource dissipation, and judicial error in the court's decision-making process. Section 5 discusses possible extensions connecting the baseline model to institutional features. Section 6 concludes with suggestions for future research. Appendix A includes the proofs omitted in the main text, while Supplementary Appendix B includes proofs of some additional results used in the main analysis.

2 Model

2.1 A patent-litigation game

We consider a litigation game between a patent holder P and a potential infringing firm Q, both assumed to be risk neutral. The game proceeds in three stages.

- Stage 1: P decides whether to sue Q. If P does not sue, the game ends and both players receive their default payoffs, which are normalized to zero. Otherwise, the game moves to stage 2.
- Stage 2: Q decides whether to defend. If Q decides not to defend, she submits to a default judgment in which the court awards P damages without holding a trial. Without loss of generality, we normalize the damage compensation value to 1. If Q defends, the game moves to a trial at stage 3.
- Stage 3: If the game proceeds to the trial stage, then both parties incur a positive participation cost $k \in [0, 1]$. In addition, the competing parties strategically spend costly resources to gather and present evidence favorable to their causes. Specifically, at the beginning of the trial, P and Q simultaneously choose the level of resources $e_P \geq 0$ and $e_Q \geq 0$, respectively. Based on the evidence and the legal environment, which reflects the merit of the patent and the presumption of its validity, the court determines whether the patent is deemed valid. We consider a probabilistic decision-making process, which we discuss in the following subsections. If the court finds the patent valid, Q pays P the damage compensation. Otherwise, she pays nothing. The game ends with the court's decision.

We normalize the expenditure function such that the costs of using resources of level e are also given by e.[15] The model assumes the American Rule regarding the payment of legal expenditures, with each party paying their own legal expenditures.[16] In Section 5.1, we

[15]A convex cost function is sufficient for the existence of an interior solution to the payoff-maximization problem. The specific functional form is beneficial for deriving a closed-form solution of the Nash equilibrium.

[16]With the English Rule, the litigation game may not have pure-strategy equilibrium for all parameter values. On the American vs English Rule, see, e.g., Massenot et al. (2021). See also the concluding section for further discussion.

extend the baseline model to allow for the possibility of out-of-court settlement between stages 2 and 3.

There are two types of costs associated with a trial: a fixed participation cost and resources spent in collecting evidence. The participation costs involve costs, such as time spent and psychological stress, that parties must endure even if they opt not to allocate resources to gather evidence. In contrast, players strategically allocate their resources to improve their chances to secure a favorable outcome. In a default judgment, parties can avoid paying any trial-related costs.

2.2 Belief about the patent validity

Following Skaperdas and Vaidya (2012), we model the court's decision-making as a process of persuasion. The alleged infringing firm, Q, can defend itself by denying infringement, challenging the validity of the patent, or both. This scenario resembles a non-bifurcated legal system with an invalidity argument as a defense. In non-bifurcated systems, such as those in the UK or Italy, there is no separation between infringement and validity proceedings. Defendants can challenge a patent's validity within infringement proceedings. Conversely, in bifurcated legal systems like those in Germany and China, infringement and validity proceedings are handled separately in distinct courts (Cremers et al., 2016; we return to this point in the concluding section.).

Our model focuses on an invalidity argument in a patent infringement litigation. Specifically, we develop a persuasion framework that focuses on seeking favorable evidence by both parties in relation to the question of validity. Assume that there are two possible states $s \in \{V, I\}$ of the world: one in which the patent is valid (V), and the other in which the patent is invalid (I). The court's prior, denoted by θ, and posterior, denoted by π, quantify its belief about the event $s = V$, before and after it considers the evidence produced during the trial, respectively.

The court's prior is a subjective assessment of the patent's validity before examining evidence. We assume that a patent's underlying merit and the presumption criterion influence the prior. The merit of a patent refers to the presumption-free belief about the state of patent's validity. We consider two presumption scenarios: one in which there is a presumption of validity (PV), and the other in which there is no presumption of validity (NP). We use a simple reduced-form representation of the prior of the following form:

$$\theta = \begin{cases} \alpha + (1-\alpha)\, m & \text{under PV} \\ m & \text{under NP} \end{cases}, \tag{1}$$

where $m \in (0, 1)$ represents the merit and $\alpha \in (0, 1)$ measures the relative weight on the presumption of validity. We can interpret the court's weighted prior under PV in the following way. A patent's validity can be assessed based on multiple criteria. One can find patent-specific free information on some criteria (of proportion $1-\alpha$) and the patent's intrinsic merit is useful in assessing validity on these grounds. On other criteria (of proportion α), no information is available and the presumption of patent's validity guides the prior assessment. It is worth noting that the effect of the presumption criterion is only limited to influencing the court's prior—it must not alter the competing parties' belief about the underlying state. Therefore, even when there is a presumption of validity, the competing parties' belief must be given by the presumption-free prior, i.e., $s = V$ with probability m.

The affine mapping in (1) is a reduced-form device: it keeps the prior in $(0, 1)$, nests the no-presumption case at $\alpha = 0$, and preserves tractability in the litigation subgame. Nonlinear alternatives can be accommodated by allowing α to depend on observables; Section 5.4 sketches state-dependent weights $\alpha(z)$. Our analyses in Section 4 show that the key comparative statics depend on the monotonicity and curvature of equilibrium trial-stage effort and the posterior belief as functions of the prior θ. Those properties are governed by the evidence technology rather than the functional form of $\theta(m)$, and so a monotone nonlinear mapping from m to θ would preserve the qualitative results while potentially shifting the merit-threshold values that determine the relative efficiency of alternative presumption criteria.

Throughout the paper, merit m is common knowledge: we do not model private information or ex ante uncertainty about m. This assumption is particularly consequential for the settlement extension developed in Section 5.1, where we introduce a pre-trial bargaining stage. Under complete information, settlement absorbs trial on the equilibrium path. If m were instead private information held by the patent holder, this bargaining stage would become a signaling game in which the settlement demand conveys information about the patent's merit. Under such informational frictions, the complete-information result that settlement absorbs trial need no longer hold: trials may arise in equilibrium as a costly device through which patent holders credibly communicate merit (Spier, 2007). These extensions are natural directions for future work.

The updating of the court's belief adheres to Bayes' Rule. Let E_i denote the evidence produced by $i \in \{P, Q\}$ during the trial. Given a prior θ, the court's posterior belief is given by

$$\pi = \Pr\left(s = V \mid E_P, E_Q\right) = \frac{\theta L^V}{(1-\theta) + \theta L^V},$$

where L^V is the likelihood ratio of patent validity based on evidence and is given by

$$L^V\left(E_P, E_Q\right) = \frac{\Pr\left(E_P, E_Q \mid V\right)}{\Pr\left(E_P, E_Q \mid I\right)}.$$

The likelihood ratio is also a subjective assessment of validity based on evidence. It is common in the literature to consider the likelihood ratio in power-law form (see, e.g., Skaperdas and Vaidya, 2012):

$$L^V(E_P, E_Q) = \left(\frac{E_P}{E_Q}\right)^{\mu},$$

where the pieces of evidence E_P and E_Q are expressed on the $(0, \infty)$ scale. The parameter $0 < \mu \leq 1$ indicates sensitivity to the evidence (see Supplementary Appendix B for more discussion). We can therefore express the posterior probability as

$$\pi(E_P, E_Q, \theta) = \frac{\theta (E_P)^{\mu}}{(1-\theta)(E_Q)^{\mu} + \theta (E_P)^{\mu}}. \tag{2}$$

2.3 Production of evidence

As in Skaperdas and Vaidya (2012), we treat evidence as the realized intensity of favorable information that a party is able to present at trial. Each party chooses a nonnegative expenditure e_i before uncertainty resolves; the draws (E_P, E_Q) are then generated by the state-conditional technology below. The court observes the evidence pair (E_P, E_Q) but does not observe (e_P, e_Q). The evidence production function depends on both the resources spent and the underlying state:

$$E_P(e_P, s) = \begin{cases} he_P & \text{with probability } f(s) \\ e_P & \text{with probability } 1 - f(s) \end{cases},$$

$$E_Q(e_Q, s) = \begin{cases} e_Q & \text{with probability } f(s) \\ he_Q & \text{with probability } 1 - f(s) \end{cases}, \tag{3}$$

where $h > 1$, $s \in \{V, I\}$, and

$$f(V) = 1 - f(I) = \gamma > \frac{1}{2}. \tag{4}$$

This evidence production function reflects how the true state positively influences the availability of favorable evidence. For example, in state V, in which the patent is valid, P is more likely (with probability $\gamma > 1/2$) to produce a higher volume of favorable evidence compared to Q , given equal resource spending. The opposite effect is observed in state I. We assume that E_P and E_Q are independent variables. This conditional independence assumption ensures tractability of the posterior aggregation via the power-law likelihood ratio. In practice, evidence-gathering activities may exhibit strategic interdependencies: for example, one party's discovery requests can reveal information that shapes the other party's evidence strategy, or a resource-constrained defendant may reduce evidence effort in response to a well-funded plaintiff's litigation posture. Section 5.4 provides a partial robustness check through a formal asymmetric-evidence variant in which only the defendant produces invalidity evidence, eliminating one source of cross-party dependence while preserving the presumption's core channel.

2.4 A probabilistic decision rule

The court uses the following probabilistic decision rule to arrive at its verdict: Choose V with probability $\pi(E_P, E_Q, \theta)$ and choose I with probability $(1 - \pi(E_P, E_Q, \theta))$. The success probabilities of P and Q of winning the trial are therefore given by $\pi(E_P, E_Q, \theta)$ and $(1 - \pi(E_P, E_Q, \theta))$, respectively.

We let U_P and U_Q denote the expected payoffs of P and Q, respectively. We assume that all parameter values are common knowledge and analyze the Bayesian Nash equilibrium of the game.

3 The equilibrium analysis

We begin our analysis at the final stage of the game. Consider the sub-game in which both players arrive at a trial. The participation cost is now sunk and P and Q simultaneously choose their expenditures (e_P, e_Q) to maximize their respective expected payoffs. As the transfer of damage compensation takes place only if P wins, the expected payoffs are as follows:

$$\begin{aligned} U_P &= \mathbb{E}_s[\pi(E_P, E_Q, \theta)] - e_P - k, \\ U_Q &= -\mathbb{E}_s[\pi(E_P, E_Q, \theta)] - e_Q - k, \end{aligned} \tag{5}$$

where $\mathbb{E}_s[\cdot]$ is the expectation operator over the probability distribution of s.

A profile of resources (e_P, e_Q) can lead to four possible events with distinct profiles of evidence (E_P, E_Q), which are (he_p, e_Q), (he_p, he_Q), (e_p, he_Q), and (e_p, e_Q). We denote these events by $event_{h1}$, $event_{hh}$, $event_{1h}$, and $event_{11}$, respectively, such that the subscript ij refers to the event in which the evidence profile is $(ie_P, je_Q), i \in \{h, 1\}, j \in \{h, 1\}$. Because E_P and E_Q are independent, we determine the state-conditional probabilities of these events using the marginal distributions, as described in (3) and (4). Specifically,

$$\Pr[event_{h1} \mid s] = \begin{cases} \gamma^2 & \text{if } s = V \\ (1-\gamma)^2 & \text{if } s = I \end{cases}; \quad \Pr[event_{hh} \mid s] = \begin{cases} \gamma(1-\gamma) & \text{if } s = V \\ \gamma(1-\gamma) & \text{if } s = I \end{cases};$$

$$\Pr[event_{1h} \mid s] = \begin{cases} (1-\gamma)^2 & \text{if } s = V \\ \gamma^2 & \text{if } s = I \end{cases}; \quad \Pr[event_{11} \mid s] = \begin{cases} \gamma(1-\gamma) & \text{if } s = V \\ \gamma(1-\gamma) & \text{if } s = I \end{cases}. \tag{6}$$

As already discussed, differently from the beliefs about contest success probabilities that are influenced by court's prior and thus affected by the presumption criterion, the competing parties' beliefs about the state of validity at the time of spending resources are solely guided by the presumption-free prior. We can therefore determine the competing parties' ex-ante belief about $event_{ij}$, denoted by q_{ij}, as

$$q_{ij} := \Pr\left[event_{ij}\right] = \Pr\left[event_{ij} \mid s = V\right] \Pr\left[s = V\right] + \Pr\left[event_{ij} \mid s = I\right] \Pr\left[s = I\right],$$

which gives us

$$\begin{aligned} q_{h1} &= \Pr\left[event_{h1}\right] = m\gamma^2 + (1-m)(1-\gamma)^2, \\ q_{hh} &= \Pr\left[event_{hh}\right] = \gamma(1-\gamma), \\ q_{1h} &= \Pr\left[event_{1h}\right] = m(1-\gamma)^2 + (1-m)\gamma^2, \\ q_{11} &= \Pr\left[event_{11}\right] = \gamma(1-\gamma). \end{aligned}$$

For notational convenience, we define $q_0 := q_{hh} + q_{11} = 2\gamma(1-\gamma)$. Then, P's expected payoff given a profile of resources (e_P, e_Q) is given by

$$\begin{aligned} U_P &= \sum_{i\in\{h,1\},j\in\{h,1\}} \pi\left(ie_p, je_Q, \theta\right) \Pr\left[event_{ij}\right] - e_P - k \\ &= \frac{q_{h1}\theta\left(he_P\right)^\mu}{(1-\theta)\left(e_Q\right)^\mu + \theta\left(he_P\right)^\mu} + \frac{q_{1h}\theta\left(e_P\right)^\mu}{(1-\theta)\left(he_Q\right)^\mu + \theta\left(e_P\right)^\mu} \\ &\quad + \frac{q_0\theta\left(e_P\right)^\mu}{(1-\theta)\left(e_Q\right)^\mu + \theta\left(e_P\right)^\mu} - e_P - k \end{aligned} \tag{7}$$

To derive the terms in the final expression of (7), we apply that $\pi\left(he_p, he_Q, \theta\right) = \pi\left(e_p, e_Q, \theta\right)$. Similarly, we can write Q's expected payoff as

$$\begin{aligned} U_Q &= -\sum_{i\in\{h,1\},j\in\{h,1\}} \pi\left(ie_p, je_Q, \theta\right) \Pr\left[event_{ij}\right] - e_Q - k \\ &= -\frac{q_{h1}\theta\left(he_P\right)^\mu}{(1-\theta)\left(e_Q\right)^\mu + \theta\left(he_P\right)^\mu} - \frac{q_{1h}\theta\left(e_P\right)^\mu}{(1-\theta)\left(he_Q\right)^\mu + \theta\left(e_P\right)^\mu} \\ &\quad - \frac{q_0\theta\left(e_P\right)^\mu}{(1-\theta)\left(e_Q\right)^\mu + \theta\left(e_P\right)^\mu} - e_Q - k. \end{aligned} \tag{8}$$

The following lemma characterizes the unique symmetric Nash equilibrium of the subgame. The proof is reported in Appendix A.

Lemma 1. *In the litigation contest, both parties incur the same expenditure, e^c, which is given by*

$$e^c = \theta(1-\theta)\mu\Gamma, \tag{9}$$

where Γ is expressed as a function of m, γ, h, μ, and θ, and is given by

$$\Gamma\left(m,\gamma,h,\mu,\theta\right)=\frac{q_{h1}h^{\mu}}{\left(\left(1-\theta\right)+\theta h^{\mu}\right)^{2}}+q_{0}+\frac{q_{1h}h^{\mu}}{\left(\left(1-\theta\right)h^{\mu}+\theta\right)^{2}}. \tag{10}$$

In this probabilistic contest, the legal expenditures are rent-seeking and both parties spend the same amount of resources in equilibrium.[17] Nevertheless, there is a possibility of learning from evidence because the court's posterior can differ from its prior. The likelihood ratio of patent validity based on evidence, $L^{V}\left(E_P,E_Q\right)$, is $(he^{*}/e^{*})^{\mu}=h^{\mu}$ and $(e^{*}/he^{*})^{\mu}=1/h^{\mu}$ in the events $event_{h1}$ and $event_{1h}$, respectively, and remains at 1 in the events $event_{hh}$ and $event_{11}$. We can describe the transition of the court's belief, from its prior to posterior, in the equilibrium path as follows:

$$\pi^{\text{posterior}}=\begin{cases}\frac{\theta h^{\mu}}{(1-\theta)+\theta h^{\mu}} & \text{with probability } q_{h1}\\ \theta & \text{with probability } q_{0}\\ \frac{\theta}{(1-\theta)h^{\mu}+\theta} & \text{with probability } q_{1h}\end{cases}. \tag{11}$$

We denote the ex-ante expected value of the posterior by π^{e}, which is given by

$$\pi^{e}=\theta\left[\frac{h^{\mu}q_{h1}}{(1-\theta)+\theta h^{\mu}}+q_{0}+\frac{q_{1h}}{\left(1-\theta\right)h^{\mu}+\theta}\right] \tag{12}$$

Recall that the function $\pi(E_P,E_Q,\theta)$ in (2) maps realized evidence and the prior into the court's posterior probability of validity. In (12), the scalar π^{e} denotes the *ex ante* expectation of that posterior, formally $\mathbb{E}_{s}\Big[\pi(E_P,E_Q,\theta)\Big]$, evaluated at the symmetric equilibrium expenditure pair $(e_P,e_Q)=(e^{c},e^{c})$ and the stochastic evidence technology (3)–(4). The scalar π^{e} depends on (γ,h,μ,θ). When we later write $\pi^{e}(\theta)$, as in (15), we emphasize its dependence on the prior, suppressing other parameters. The ex-ante expected payoffs of P and Q at the trial are

$$\mathbb{E}\left(U_P\right)=\pi^{e}-e^{c}-k,$$
$$\mathbb{E}\left(U_Q\right)=-\pi^{e}-e^{c}-k.$$

Next, we analyze stage 2. Consider Q's decision to defend. She can either pay the compensation value of 1 or defend a trial, in which case, her expected payoff is $\mathbb{E}\left(U_Q\right)$. Therefore, Q defends if $-1\leq\mathbb{E}\left(U_Q\right)$, or, equivalently, if

$$\pi^{e}\leq 1-e^{c}-k. \tag{13}$$

[17] This is a well-known result from the large rent-seeking literature building on Tullock (1978; 1997), also when applied to litigation (Guerra et al., 2019b; Friehe and Wohlschlegel, 2019; Massenot et al., 2021).

Finally, we analyze stage 1. Consider P's decision to litigate. P receives a zero payoff if she does not litigate. Her payoff from litigation depends on Q's response. In particular, if (13) is not satisfied, then default judgment occurs and P receives a payoff of one. If (13) is satisfied, Q defends a trial, in which case P has a positive expected payoff only if $\mathbb{E}(U_P) \geq 0$, or equivalently, if $\pi^e \geq e^c + k$. Therefore, P litigates in the following two situations: one in which Q submits to a default judgment, which occurs if $\pi^e \geq 1 - e^c - k$; the other in which Q responds by defending a trial, which occurs if $e^c + k < \pi^e < 1 - e^c - k$. The first situation presents a first-mover advantage that P has in this sequential game. Combining the two conditions, we find that P litigates if

$$\pi^e \geq \min\{e^c + k, 1 - e^c - k\}. \tag{14}$$

We can now fully characterize the equilibrium outcome based on the relationship between the expected posterior π^e and the cost-based thresholds $(e^c + k)$ and $(1 - e^c - k)$. Three different regimes can arise in equilibrium:

- **Litigation trial**: P litigates and Q defends a trial. This arises if $e^c + k \leq \pi^e \leq 1 - e^c - k$.
- **Default judgment**: P litigates and Q pays the damage compensation. This arises if (13) does not hold.
- **No litigation**: P does not litigate. This arises if (14) does not hold.

The following proposition characterizes the equilibrium outcome of the litigation game. The proof follows straightforwardly from the preceding discussion.

Proposition 1. *The equilibrium regimes of the litigation game are characterized as follows:*

(a) If $\pi^e < \min\{e^c + k, 1 - e^c - k\}$*, then the no-litigation regime prevails.*

(b) If $\pi^e > 1 - e^c - k$*, then the default-judgment regime prevails.*

(c) If $e^c + k \leq \pi^e \leq 1 - e^c - k$*, then the litigation-trial regime prevails. This range can be vacuous if* $e^c + k > 1/2$*.*

The intuition behind Proposition 1 is straightforward. The expected posterior π^e reflects how players perceive P's chance of winning a trial. Therefore, the incentives of P and Q to engage in a trial are increasing and decreasing in π^e, respectively. The litigation-trial regime prevails, if at all, when the expected posterior falls within the two cost-based threshold values $(e^c + k)$ and $(1 - e^c - k)$.

4 Effects of the presumption of validity

The presumption criterion induces a bias to the prior, which is otherwise determined by the underlying merit of a patent. The merit influences the outcome of the litigation game in two different ways: first, through the prior; and second, by moderating the expected return to investment. The first effect is qualitatively similar to the effect of the bias induced by the presumption of validity—and we are particularly interested to delineate this effect. However, by simply measuring the incremental effect of m, we will also capture the second effect.

To circumvent this challenge, we adopt an indirect approach. We fix the merit m and study how changing a generic prior θ affects various derived parameters of our model. By delineating the effect of a generic prior after controlling for the merit's effect, we get to measure how the presumption-induced bias to the prior would affect the outcome of the game.[18]

To this end, consider first the effect of a generic prior θ on the expected posterior, π^e, and the resources spent, e^c, for a fixed value of m. It follows from (12) that $d\pi^e/d\theta > 0$, implying that the expected posterior π^e is increasing in θ. The difference between the expected posterior π^e and the prior can be expressed as

$$\pi^e(\theta) - \theta = \theta(1-\theta)(h^\mu - 1)\left[\frac{q_{h1}}{(1-\theta)+\theta h^\mu} - \frac{q_{1h}}{(1-\theta)h^\mu + \theta}\right], \tag{15}$$

from which it follows that the expected posterior coincides with the prior at $\theta = 0$, 1, and at some $\widehat{\theta}$ that satisfies $q_{h1}/((1-\theta)+\theta h^\mu) = q_{1h}/((1-\theta)h^\mu + \theta)$. Further, whenever $\widehat{\theta} \in (0,1)$, the posterior is above the prior for $\theta \in (0, \widehat{\theta})$ and below the prior for $\theta \in (\widehat{\theta}, 1)$.

The resource-spending level e^c changes non-monotonically with respect to θ; $e^c = 0$ at $\theta = 0, 1$, and $e^c > 0$ for $\theta \in (0,1)$. In Lemma A.1 in Appendix A, we show that the threshold e^c is increasing in $\theta \in [0, 1/(h^\mu + 1)]$ and decreasing in $\theta \in [h^\mu/(h^\mu+1), 1]$. Further, if $h^\mu \leq 2$, then e^c is concave, and consequently, $1 - e^c$ is convex.

In the following analysis, in the interest of tractability, we assume $h^\mu \leq 2$.[19] This assumption implies that e^c has a unique maximum at some prior $\theta \in [1/(h^\mu+1), h^\mu/(h^\mu+1)]$.

Assumption 1. *$h^\mu \leq 2$.*

[18] Note that we can achieve similar findings by studying the derivatives of π^e and e^c with respect to α while keeping the values of m fixed. However, these derivatives involve complex algebraic expressions, hence becoming less tractable. In contrast, our approach enables us to interpret the findings in terms of the shifting effects of the prior.

[19] In Supplementary Appendix B, we discuss how the results would be affected if this assumption is violated.

4.1 The equilibrium regimes

We first examine how shifting the prior between the two presumption scenarios, PV and NP, affects the equilibrium regimes. Despite the non-monotonic effect of the prior on the resources spent during the trial, we find that the expected trial payoffs of P and Q change monotonically with respect to θ. Consequently, there exist threshold values for the prior delineating players' willingness to engage in a trial. The following lemma documents the observation, with the proof reported in Appendix A.

Lemma 2. *For any given $k \geq 0$, there exists thresholds $\underline{\theta} \geq 0$ and $\overline{\theta} \geq 0$ such that $\pi^e \geq e^c + k$ if and only if $\theta \geq \underline{\theta}$, and $\pi^e \leq 1 - e^c - k$ if and only if $\theta \leq \overline{\theta}$. If $k = 0$, $\underline{\theta} = 0$ and $\overline{\theta} = 1$. Further, $\underline{\theta}$ increases with k and $\overline{\theta}$ decreases with k.*

It follows from Lemma 2 that the relationship between the expected posterior π^e and the cost-based thresholds $(e^c + k)$ and $(1 - e^c - k)$ can appear in three possible forms, as illustrated in Figure 1. In the first form, occurring if $k = 0$ and displayed in the left panel of Figure 1, $\pi^e \in [e^c, 1 - e^c]$ for all θ, and therefore only the litigation-trial regime prevails. In the second form, displayed in the middle panel of Figure 1, $0 < \underline{\theta} \leq \overline{\theta} < 1$, and there are three different regimes in equilibrium—the no-litigation regime for $\theta \in [0, \underline{\theta})$; the litigation-trial regime for $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$; and the default-judgment regime for $\theta \in \left(\overline{\theta}, 1\right]$. Finally, in the third form, displayed in the right panel of Figure 1, $0 < \overline{\theta} < \underline{\theta} < 1$, and there are two different regimes in equilibrium—the no-litigation regime for $\theta \in \left[0, \overline{\theta}\right)$; and the default-judgment regime for $\theta \in \left(\overline{\theta}, 1\right]$.

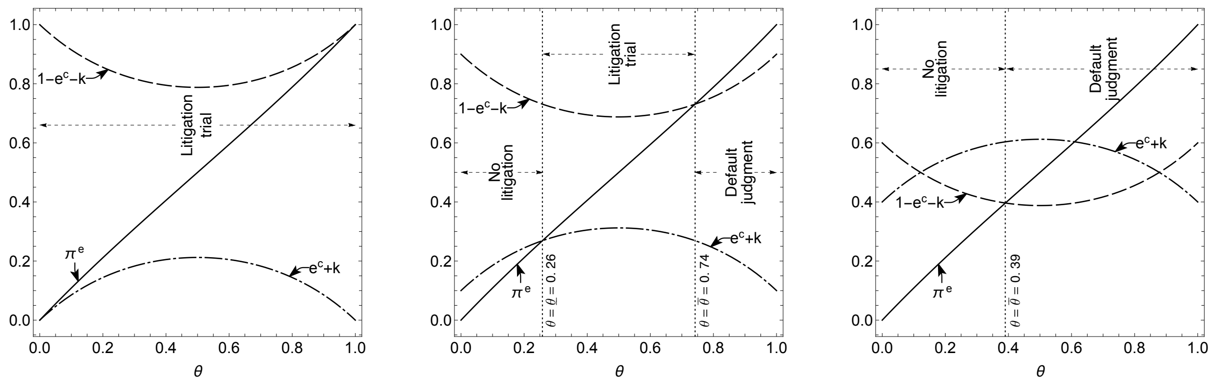

Figure 1: The expected posterior, the cost-based thresholds, and the equilibrium regimes against θ

Notes. All plots in Figure 1 consider the parameter values: $\mu = 0.9$, $h = 2$, $m = 0.5$, $\gamma = 0.75$. Further, we consider $k = 0$, 0.1, and 0.4, in the left-panel, middle-panel, and right-panel diagrams respectively. The continuous curve, the dot-dashed curve, and the dashed curve present π^e, $e^c + k$, and $1 - e^c - k$, respectively.

We can now easily compare the equilibrium regime between the two presumption scenarios—PV and NP. The presumption of validity moves the prior θ upward, i.e., from m to $\alpha + (1-\alpha)\,m$, which takes a value in $(m,1)$. As the no-litigation regime prevails for $\theta \in \left[0, \min\left\{\underline{\theta}, \overline{\theta}\right\}\right)$ and the default-judgment regime prevails for $\theta \in \left(\overline{\theta}, 1\right]$, it follows that an increase in θ will reduce the possibility of the no-litigation regime and increase the possibility of the default-judgment regime. The effect on the possibility of the litigation-trial regime is ambiguous: it depends on the relative extent of the bias in comparison to the thresholds $\underline{\theta}$ and $\overline{\theta}$.

The following proposition summarizes the above observations. The proof trivially follows from the preceding discussion.

Proposition 2. *Consider, as a point of comparison, NP as the default scenario. The introduction of the presumption of validity expands the set of parameter values for which the default-judgment regime prevails in equilibrium and shrinks the set of parameter values for which the no-litigation regime prevails in equilibrium. Further, the set of parameter values for which the litigation-trial regime prevails in equilibrium can either expand or shrink.*

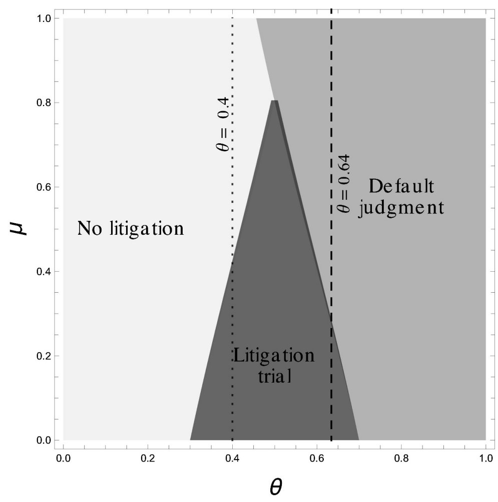

Figure 2: The equilibrium regimes in the (θ, μ) space

Notes. Figure 2 considers the parameter values: $h = 2$, $\gamma = 0.9$, $m = 0.4$ and $k = 0.3$. The equilibrium regimes are plotted in the (θ, μ) space. The dotted and the dashed line represent the priors $\theta = 0.4$ and $\theta = 0.64$, respectively.

Figure 2 plots the equilibrium regimes in the (θ, μ) space. To understand the findings of Proposition 2, consider a patent of merit $m = 0.4$ and $\alpha = 0.4$. If there is no presumption, the prior is $\theta = m = 0.4$, which is represented by the dotted line in Figure 2. At $\theta = 0.4$, the litigation-trial regime exists for low values of μ. With the presumption of validity, the prior is $\theta = 0.4 + 0.6 \times 0.4 = 0.64$, which is represented by the dashed line in the figure. At $\theta = 0.64$, the default-judgment regime prevails over a larger range of values of μ and the no-litigation regime ceases to occur.

Proposition 2 shows that the presumption of validity affects the existence of the litigation-trial regime in equilibrium in an ambiguous way. To explore further, we next focus on two interesting features of the litigation-trial regime. First, a litigation trial is costly because resources are dissipated. Second, there is a scope for learning through evidence produced in the trial. The learning moderates the possibility of making errors of judgment, i.e., rejecting the validity of a valid patent and accepting the validity of an invalid patent during a trial. Subsection 4.2 examines aggregate resource dissipation, while Subsection 4.3 analyzes judgment error. When we compare PV to NP below, we restrict to parameter values for which the litigation-trial regime prevails in equilibrium under both presumption scenarios—so that both objects are evaluated along the same equilibrium path in which a trial occurs.

4.2 Resource dissipation

The total resource dissipation is given by

$$R = 2e^c + 2k = 2\theta\left(1-\theta\right)\mu\varGamma + 2k.$$

The presumption criterion affects R only through the prior θ. We first study how a generic prior θ affects R for a fixed value of m. It follows from Lemma A.1 that R is increasing in $\theta \leq 1/\left(h^\mu + 1\right)$ and decreasing in $\theta \geq h^\mu/\left(h^\mu+1\right)$. Further, under Assumption 1, R has its maximum at a unique θ, which we denote by θ^R. Specifically,

$$\theta^R := \arg\max_{\theta\in[0,1]} R \tag{16}$$

Under Assumption 1, it follows from the first-order condition of (16) that θ^R uniquely solves the following:

$$\left[\frac{q_{h1}h^\mu\left(h^\mu+1\right)}{\left(\left(1-\theta\right)+\theta h^\mu\right)^3}\left(\frac{1}{h^\mu+1}-\theta\right)+2q_0\left(\frac{1}{2}-\theta\right)+\frac{q_{1h}h^\mu\left(h^\mu+1\right)}{\left(\left(1-\theta\right)h^\mu+\theta\right)^3}\left(\frac{h^\mu}{h^\mu+1}-\theta\right)\right]=0. \tag{17}$$

The following lemma documents the relationship between a patent's merit m and θ^R.

Lemma 3. *Fix m and consider R as a function of a generic prior θ. Under Assumption 1, R is uniquely maximized at some $\theta^R \in (0,1)$. Further, $\theta^R < 1/2$ if $m > 1/2$, and $\theta^R > 1/2$ if $m < 1/2$. And, $\theta^R = m$ if $m = 1/2$.*

Figure 3 plots θ^R against m for a certain parametric specification. Suppose that the true state does not influence evidence production, i.e., $h = 1$. Then, it follows from (10) that $\varGamma = 1$ and $R = 2\theta\left(1-\theta\right)\mu + 2k$, which is maximized at $\theta = 1/2$. Therefore, for every m, θ^R remains at $1/2$; in other words, if the true state did not influence the availability of favorable evidence, both parties expend their resources to the full extent when the court deems both states equally likely.

To understand the findings of Lemma 3, consider what might happen if $h > 1$. We first argue for the case $m > 1/2$. As $h > 1$, P finds it easier to produce favorable evidence compared to Q. The asymmetry discourages Q to expend resources, and, consequently, both parties' contest effort levels decrease. This discouragement effect can be partially mitigated if the court perceived the state of patent validity to be less likely, which would give Q an incentive to expend resources. This is the reason why θ^R, the resource-dissipation-maximizing prior value, must be less than 1/2 whenever $m > 1/2$. Similarly, for $m < 1/2$, the asymmetric cost of evidence production discourages P to expend resources, and this discouragement effect can only be partially compensated if the court's prior is favorably biased toward the state of patent validity, resulting in $\theta^R > 1/2$.

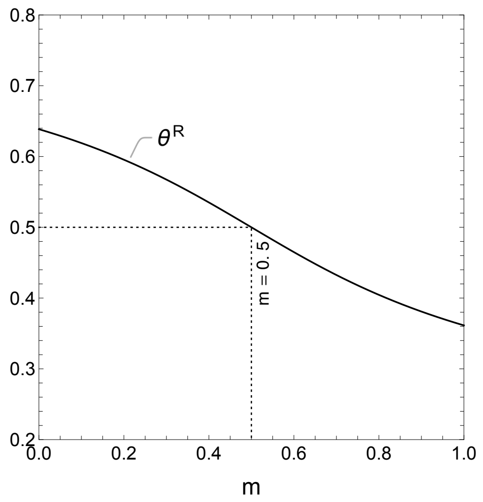

Figure 3: θ^R against m

Notes. Figure 3 considers the following parameter values: $\mu = 1$, $h = 2$, $\gamma = 0.9$.

Note that the presumption of validity not only changes the value of R, but also the type of the equilibrium regime. If, for some parametric specification, there is litigation-trial regime in equilibrium in one presumption scenario but not the other, then comparing resource dissipation between the two scenarios is straightforward. In contrast, if there is litigation trial in equilibrium under both PV and NP, then measuring the effect of the presumption on R is more complex.

To look into the effect of the presumption of validity, we now focus on the parameter space for which there will be litigation trial under both PV and NP and compare R between the two scenarios as m changes. Assume, without loss of generality, NP to be the default scenario and consider patents with $m > 1/2$. Then, $\theta^R < 1/2 < m$. The presumption of validity moves the value of the prior upward in the range $(m, 1)$. As R is decreasing in $\theta \in [\theta^R, 1]$, the presumption of validity will only reduce R. For $m < 1/2$,

$m < 1/2 < \theta^R$. The effect on R is ambiguous; it can increase or decrease, depending on values of α and m, which determine to what extent the presumption-driven prior θ increases from m. Using the fact that R is concave under Assumption 1, the following proposition shows that the aggregate resource dissipation R is strictly lower under NP than under PV if and only if m is below a threshold lower than 1/2.

Proposition 3. *Consider the range of parameter values for which the litigation-trial regime prevails in equilibrium in both the presumption scenarios, PV and NP. Further, consider, as a point of comparison, NP as the default scenario. There exists a threshold $m^R_{PV} \in (0, 1/2)$ such that the introduction of PV will increase (decrease) the aggregate resource dissipation R if m is less (greater) than m^R_{PV}.*

The intuition behind Proposition 3 is straightforward. As m deviates from 1/2 in either direction, one of the two parties finds it easier to produce evidence, creating an imbalance that generally discourages both parties from investing. For high-merit patents, the presumption of validity exacerbates the existing imbalance. In contrast, for low-merit patents, the presumption partially mitigates the imbalance. Therefore, the threshold merit level at which the presumption can effectively neutralize the discouraging effects caused by asymmetric merit is always below 1/2.

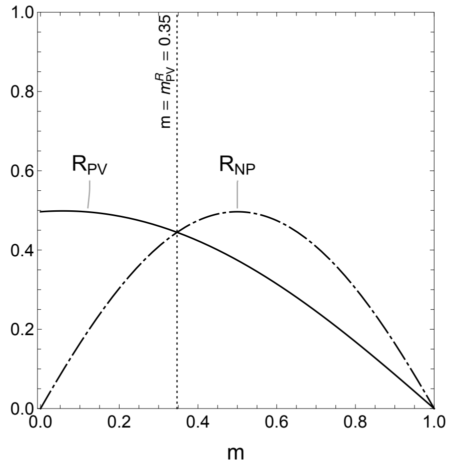

Figure 4: R against m

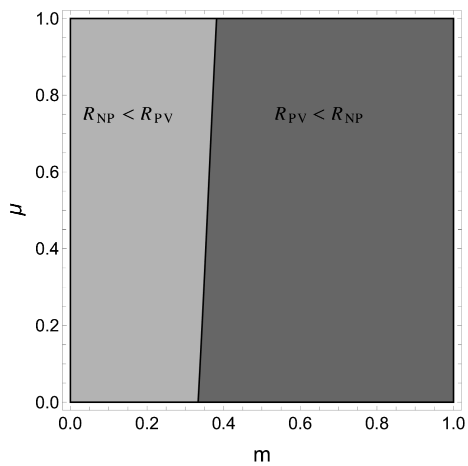

Figure 5: Comparison of R between PV and NP in the (m, μ) space

Notes. Figure 4 considers the parameter values: $k = 0$, $\mu = 1$, $h = 2$, $\gamma = 0.9$, and $\alpha = 0.5$. R_{PV} (the continuous curve) and R_{NP} (the dot-dashed curve) denote the resource dissipation R under $\theta = \theta_{PV}$ and $\theta = m$, respectively and we have $R_{NP} < R_{PV}$ if $m < m^R_{PV} = 0.35$. Figure 5 considers the same parameter values (relaxing μ) and compares R_{PV} with R_{NP} in the (m, μ) space.

In Figure 4, we plot R against m under PV and NP. Figure 5 compares R between PV and NP in the (m, μ) space. In Figures 4 and 5, we consider $k = 0$ so that the litigation-trial regime prevails in equilibrium for all possible priors under both PV and NP.

4.3 Error of judgment

An error of judgment in the court's decision-making occurs if the outcome of the litigation game results in either rejecting a patent's validity when it is valid (equivalent to a false negative or type-II error in statistical binary classification), or accepting a patent's validity when it is invalid (equivalent to a false positive or type-I error). The aggregate probability of making an error of judgment in litigation is given by[20]

$$\begin{aligned} E &= \Pr\left[P \text{ wins } \cap s = I\right] + \Pr\left[Q \text{ wins } \cap s = V\right] \\ &= \Pr\left[P \text{ wins } \mid s = I\right] \Pr\left[s = I\right] + \Pr\left[Q \text{ wins } \mid s = V\right] \Pr\left[s = V\right] && (18) \\ &= m + \sum_{i \in \{h,1\}, j \in \{h,1\}} \pi\left(ie_p, je_Q, \theta\right) \begin{bmatrix} (1-m)\Pr\left[event_{ij} \mid s = I\right] \\ -\, m \Pr\left[event_{ij} \mid s = V\right] \end{bmatrix} && (19) \end{aligned}$$

Using (2) and the fact that $e_P = e_Q = e^c$ in equilibrium, we can replace $\pi\left(ie_p, je_Q, \theta\right)$ by $\theta i^\mu / [(1-\theta) j^\mu + \theta i^\mu]$. Further, in $event_{hh}$ and $event_{11}$, we have $i = j$, and it follows that $\pi\left(ie_p, je_Q, \theta\right) = \theta$. Using the state-conditional probabilities of various events from (6), (19) further reduces to the following expression:

$$\begin{aligned} E = m &+ \frac{\theta h^\mu \left[(1-m)(1-\gamma)^2 - m\gamma^2\right]}{(1-\theta) + \theta h^\mu} \\ &+ \frac{\theta\left[(1-m)\gamma^2 - m(1-\gamma)^2\right]}{(1-\theta) h^\mu + \theta} + 2\theta\left[(1-2m)\gamma(1-\gamma)\right]. && (20) \end{aligned}$$

As shown in (18), E is a weighted sum of two conditional probabilities, measuring chances of a false positive ($\Pr\left[P \text{ wins } \mid s = I\right]$) and false negative ($\Pr\left[Q \text{ wins } \mid s = V\right]$). Further, the weights are not constant across merits. The weight associated with a false positive is high for low-merit patents and low for high-merit patents (see subsection 5.2 for further discussion). The following lemma documents some useful properties of E.

Lemma 4. *Fix m and consider E as a function of a generic prior θ. Then,*

(i) There exists thresholds $\underline{m} < 1/2 < \overline{m}$ such that for any $m \leq \underline{m}$, E is increasing in $\theta \in [0,1]$, and for any $m \geq \overline{m}$, E is decreasing in $\theta \in [0,1]$.

(ii) Further, for $m \leq 1/2$, E is increasing in θ for all $\theta \in [1/2, 1]$ and for $m \geq 1/2$, E is decreasing in θ for all $\theta \in [0, 1/2]$. It follows that for $m = 1/2$, E reaches its minimum at $\theta = 1/2$.

[20] Here and in what follows, we slightly ease the notation by denoting the measure of judgment error as E without any subscript. Previously, we used E_i with a subscript i to indicate the evidence produced by party i.

Part (i) of the above lemma illustrates the effect of the presumption criterion in clear terms when m takes sufficiently high or low values. For example, for $m \geq \overline{m}$, the presumption of validity only reduces an error of judgment by shifting the prior upward. The opposite effect will be realized for $m \leq \underline{m}$. For intermediate values of m, the analysis becomes complex.

A change in θ, after controlling for merit m, affects E in two distinct ways. The first is a *direct* effect. An increase in the prior θ leads to a qualitatively similar increase in the posterior π, which raises the probability of false positive and reduces the probability of a false negative. The second is an *indirect* effect. When θ gets close to $1/2$, because of the high intensity of competition, both parties spend a high volume of resources in unearthing new evidence. The probabilities of both a false positive and a false negative decrease with high evidence-seeking incentives.

To understand the direction of the combined effect, consider first the case of a low-merit patent, with m below $1/2$. Because a low-merit patent has higher weight on the false positive than on the false negative, the direct effect on E described above is increasing in θ. In contrast, the indirect effect due to evidence-seeking incentive does not follow a monotone path. As θ approaches $1/2$, the indirect effect dampens E, but then raises it as θ moves further away from $1/2$. Together, for sufficiently low values of m, the direct effect dominates the indirect because of the high weight on the conditional probability of false positive, and E increases with θ over its full range. For m close to $1/2$, the indirect effect can dominate the direct effect and E might be decreasing in $\theta \in (m, 1/2)$. However, for $\theta > 1/2$, both effects move in the same direction, which explains the second part of the above lemma. In the case of a high-merit patent with $m > 1/2$, the indirect effect works the same way as above but the direct effect works in the opposite direction. This is because a high-merit patent puts higher weight on the false negative than on the false positive, and an increase in θ reduces the probability of a false negative and raises the probability of a false positive.

Next, to study the effect of the presumption of validity, we vary m and compare E between the two scenarios, PV and NP. Because a low-merit patent ($m < 1/2$) puts more weight on the probability of a false positive than a high-merit patent ($m > 1/2$) does, the direct effect of the presumption bias typically increases the error of judgment by a higher margin for a low-merit patent. However, when m is below but close to $1/2$, the evidence-seeking incentive is high even without the presumption bias, and the marginal effect of increasing the competition intensity by biasing the prior is relatively low. Thus, the positive indirect effect of the bias by increasing the evidence-seeking incentive is always dominated by the negative direct effect of increasing the probability of a false positive for all patents with $m < 1/2$. The following proposition shows that for all m below a threshold that is weakly higher than $1/2$, PV is always associated with a higher error of judgment compared to NP.

Proposition 4. *Consider the range of parameter values for which the litigation-trial regime prevails in equilibrium in both the presumption scenarios, PV and NP. Further, consider, as a point of comparison, NP as the default scenario. There exists a threshold $m_{PV}^E \in (1/2, 1)$ such that the introduction of PV will increase the error of judgment E if m is less than m_{PV}^E.*

It is worth noting that unlike Proposition 3, Proposition 4 does not provide a necessary and sufficient threshold-based result for comparing E between the two regimes. This is because the two effects of the presumption bias described earlier, the direct effect of changing probabilities of a false positive and false negative and the indirect effect of changing evidence-seeking incentive, can move at different rates causing E to change in a non-monotone way.

Figures 6 and 7 illustrate the proposition's findings using a numerical example. In Figure 6, we plot E against m under PV and NP. Figure 7 compares E between PV and NP in the (m, μ) space. We consider $k = 0$ so that the litigation-trial regime prevails in equilibrium for all possible priors under both PV and NP.

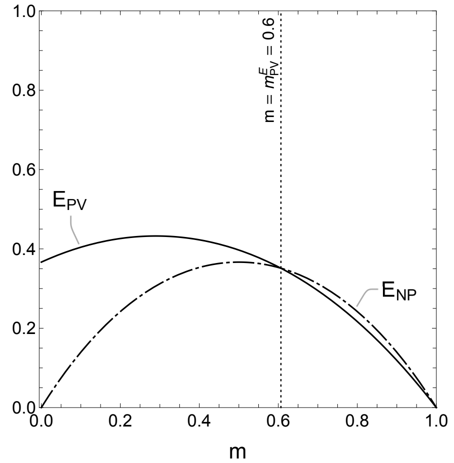

Figure 6: E against m

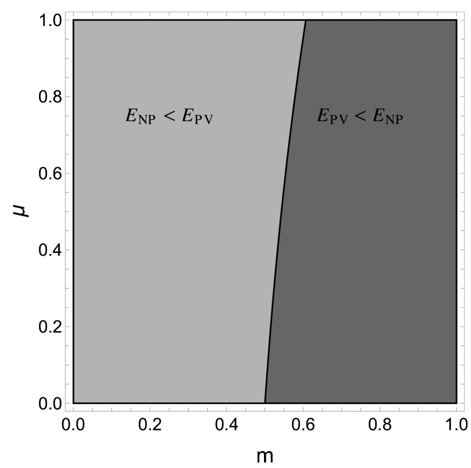

Figure 7: Comparison of E between PV and NP

Notes. Figure 6 considers the parameter values: $k = 0$, $\mu = 1$, $h = 2$, $\gamma = 0.9$, and $\alpha = 0.5$. E_{PV} (the continuous curve) and E_{NP} (the dot-dashed curve) denote the error value E, computed at the priors $\theta = \theta_{PV}$ and $\theta = m$, respectively and we have $E_{NP} < E_{PV}$ if $m < m_{PV}^E = 0.6$. Figure 7 considers the same parameter values (relaxing μ), and compares E_{PV} with E_{NP} in the (m, μ) space.

Subsections 4.2 and 4.3 study resource dissipation R and judicial error E as two features of the litigation-trial regime when a trial lies on the equilibrium path (cf. Propositions 3 and 4). In subsection 5.1 we allow an out-of-court settlement stage between stages 2 and 3; there we show how π^e and e^c govern bargaining as threat-point primitives and we clarify how settlement changes the interpretation of trial-stage expenses.

5 Discussion

This section discusses model extensions that help connect the baseline framework to institutional features of patent disputes. We begin with out-of-court settlement, then discuss how heterogeneous economic stakes scale settlement transfers while preserving the baseline comparative statics, and we indicate additional extensions for future revisions.

5.1 Settlement and threat-point effects

We extend the baseline game by allowing an out-of-court settlement stage between stage 2 (the defendant's decision to defend) and stage 3 (the trial). In our complete-information benchmark, introducing settlement has a sharp implication: disputes that reach the post-defense bargaining node settle on the equilibrium path, and realized trials disappear except at knife-edge indifference cases. This implication is standard in the litigation-settlement literature when no friction such as asymmetric information, asymmetric beliefs, or indivisibilities is present (see Reinganum and Wilde, 1986; Spier, 1992, 2007; for patent applications, see Meurer, 1989; Crampes and Langinier, 2002). This preserves Sections 3 and 4 as characterizations of continuation trial values and comparative-static primitives, while changing equilibrium-path implementation through bargaining.

Formally, the game is modified as follows: relative to the baseline timeline in Section 2, we insert a settlement stage between stage 2 (defend or default) and stage 3 (trial), keeping the rest of the game form unchanged. Specifically, if Q decides to defend at stage 2, the patent holder P makes a take-it-or-leave-it settlement demand $S \in [0, 1]$ before any evidence expenditure is sunk. If Q accepts, the game ends with a transfer of S from Q to P; otherwise, the game proceeds to trial. We characterize the SPNE by backward induction in the order stage 3 $\to$ stage 2.5 $\to$ stage 2 $\to$ stage 1, where trial continuation values determine bargaining outcomes at the settlement stage.

We maintain the same information structure as in the baseline: all primitives are common knowledge and both parties evaluate continuation trial payoffs using (12) and (9). For a given prior θ, the continuation values from rejecting settlement are the expected trial-stage payoff:

$$V_P^T(\theta) = \pi^e(\theta) - e^c(\theta) - k, \quad V_Q^T(\theta) = -\pi^e(\theta) - e^c(\theta) - k.$$

Therefore, at the settlement stage, Q accepts an offer S if and only if $V_Q^T(\theta) \leq -S$, or, equivalently,

$$S \leq \pi^e(\theta) + e^c(\theta) + k =: T(\theta), \tag{21}$$

where $T(\theta)$ determines Q's trial-stage threat-point magnitude. Because P has full bargaining power, she chooses the largest acceptable transfer subject to $S \in [0, 1]$, which yields $S^*(\theta) = \min\{1, T(\theta)\}$. Since $S^*(\theta) > 0$ for $\theta \in (0,1)$, P strictly prefers suing to not suing and therefore always litigates in equilibrium.

Proposition 5. *Consider the extended game with settlement at stage 2.5, in which P makes a take-it-or-leave-it offer. The unique equilibrium settlement payment is*

$$S^*(\theta) = \min\{1, T(\theta)\} = \min\{1, \pi^e(\theta) + e^c(\theta) + k\}.$$

Settlement occurs in equilibrium whenever Q defends. Moreover, P strictly prefers suing to not suing for every $\theta \in (0,1)$, while Q weakly prefers defending to default judgment, with strict preference if $T(\theta) < 1$. The equilibrium settlement payment is weakly higher under PV than under NP, with strict inequality whenever $T(m) < 1$.

Proposition 5 brings the trial subgame into a bargaining environment. Introducing settlement, however, adds more than a mechanical replacement of trials by agreements: the presumption affects equilibrium transfers through the same trial-stage features, π^e and e^c, which serve as threat-point primitives governing the settlement payment and the sue/no-sue margin. If settlement is accepted at Stage 2.5, trial expenditures are not incurred on the equilibrium path, so the interpretation of resource dissipation differs from the baseline: e^c and k enter as shadow values through the continuation payoffs V_P^T and V_Q^T rather than as realized costs. Relative to the three-regime classification in Section 3, the no-litigation and litigation-trial regimes are absorbed by settlement, while the default-judgment region survives only as an indifference case when $T(\theta) = 1$.

Proposition 5 has two substantive implications. First, the presumption shifts bargaining leverage toward the patent holder, enabling extraction of a larger out-of-court transfer even though no trial takes place on the equilibrium path. The increase in the threat-point magnitude $T(\theta) = \pi^e(\theta) + e^c(\theta) + k$ operates through two channels: a *winning-probability channel*, whereby π^e is strictly increasing in θ, and an *evidence-cost channel*, whereby e^c is non-monotonic in θ (Lemma A.1). Although these channels can partially offset for high-merit patents (where e^c declines with θ), Lemma A.2 guarantees that their sum is strictly increasing, so the net effect on the settlement transfer is unambiguously positive. Second, because P always sues and Q always defends (Proposition 5), the three-regime structure of Proposition 1 collapses into a single settlement regime. In particular, the no-litigation regime, in which weak patents go unasserted, vanishes entirely once settlement is available. The presumption's effect on equilibrium type (Proposition 2) is thus subsumed: the policy-relevant margin shifts from *whether* disputes arise to *how large* the settlement transfer is. For priors at which $T(\theta) < 1$, the presumption strictly inflates S^*; for priors at which $T(\theta) \geq 1$, the cap at full damages already binds and the presumption has no additional bite on the transfer. This settlement channel formalizes how the presumption can function as a credible enforcement threat that raises the cost of contested technologies even absent any courtroom proceeding (Lichtman and Lemley, 2007), and it underscores that the welfare incidence of the presumption runs primarily through bargaining outcomes in environments where settlement is the prevailing mode of dispute resolution.

The take-it-or-leave-it protocol by P is a standard reduced-form device in the settlement literature because it yields a closed-form benchmark while preserving the key threat-point logic (Reinganum and Wilde, 1986; Meurer, 1989). An alternative is Nash bargaining over the same disagreement point (V_P^T, V_Q^T). With bargaining weight $\beta \in (0,1)$ for P, the weighted Nash bargaining solution yields a settlement payment $S_\beta^{NB}(\theta) = \pi^e(\theta) + (2\beta - 1)(e^c(\theta) + k)$. The take-it-or-leave-it outcome corresponds to $\beta = 1$; the symmetric Nash case $\beta = 1/2$ gives $S_{1/2}^{NB}(\theta) = \pi^e(\theta)$, splitting the avoided trial surplus equally. Under any $\beta \in (0,1)$, the qualitative conclusions of Proposition 5 carry over: settlement occurs on the equilibrium path, P sues, Q weakly prefers defending to default judgment, and the presumption raises the equilibrium transfer because $S_\beta^{NB}(\theta)$ is increasing in θ (which follows from Lemma A.2; see Appendix A).

In practice, parties sometimes settle only after incurring discovery-style costs; we discuss post-discovery settlement timing as a future extension in the concluding section.

5.2 Welfare accounting with a market-impact wedge

The framework also helps interpret how enforcement can carry broader market consequences. Private transfers, whether realized through default judgment or settlement, are not themselves social losses, but the prospect of enforcement can distort behavior by delaying adoption, deterring entry, or raising the effective cost of using a technology (Farrell and Shapiro, 2008; Lemley and Shapiro, 2005). To capture these forces, consider a reduced-form welfare accounting that augments procedural costs and adjudication accuracy with a downstream market wedge. Recall that in the baseline model (no settlement possibility), Q defends only when $T(\theta) \leq 1$ (equation (13)), and accepts the default judgment if $T(\theta) > 1$. In the settlement extension, $S^*(\theta) = \min\{1, T(\theta)\}$ is the equilibrium settlement payment. Hence $S^*(\theta)$ serves as a unified measure of the defendant's payment obligation across both model versions.

Let $L_{FP} \geq 0$ and $L_{FN} \geq 0$ denote the social losses from a false positive (upholding an invalid patent) and a false negative (invalidating a valid patent), respectively, and let $\Lambda \geq 0$ capture the intensity with which enforcement threats operate like a "tax" on business activity and technology adoption. A reduced-form expected social-loss criterion is

$$\mathcal{L} = R + L_{FP}\,\Pr(FP) + L_{FN}\,\Pr(FN) + \Lambda\, S^*(\theta), \tag{22}$$

where R is resource dissipation (subsection 4.2), $\Pr(FP)$ and $\Pr(FN)$ are the equilibrium false-positive and false-negative probabilities whose sum equals the aggregate error E (subsection 4.3), and $S^*(\theta)$ is the payment obligation. Under settlement, trial expenditures are not realized on the equilibrium path, but the same objects π^e and e^c govern the threat point that determines $S^*(\theta)$ and hence the market-impact wedge.

Holding fixed $(L_{FP}, L_{FN}, \Lambda)$, the desirability of a stronger presumption depends on how it shifts adjudication accuracy and the payment obligation $S^*(\theta)$. In the terminology of Lichtman and Lemley (2007), the $\Lambda\, S^*(\theta)$ term captures how the presumption can enable patent holders to "tax legitimate business activity" through credible enforcement threats. The resource-dissipation term R reflects the countervailing benefit that a stronger presumption can dampen costly litigation when patent merit is not in serious doubt. Because Proposition 5 implies that the equilibrium payment obligation is weakly higher under the presumption of validity than without it, a larger Λ amplifies the welfare cost of the presumption through the market impacts, tilting the overall comparison against the presumption in environments where enforcement threats are a first-order concern for downstream market participants.

Equation (22) is a reduced-form device, not a full product-market model. Settings in which injunctive relief or other exclusionary remedies can be granted with limited evidence correspond to a large L_{FP}, reflecting that erroneous enforcement may generate substantial downstream losses, and may also elevate Λ by making enforcement threats more potent ex ante (Farrell and Shapiro, 2008). The U.S. Supreme Court's decision in *eBay Inc. v. MercExchange, L.L.C.*, 547 U.S. 388 (2006), which replaced the near-automatic injunction standard with a four-factor equitable test, underscores these concerns. We do not model preliminary relief or market dynamics explicitly; the mapping clarifies how the presumption's effects on error probabilities and threat points could translate into welfare under different market environments.

5.3 Heterogeneous stakes and prize scaling

We next relax the unit-normalization of stakes. In practice, policy concern about the presumption extends not only to *whether* weak patents can extract transfers, but also to *how large* those transfers are in high-stakes environments.

Suppose the adjudicated damage transfer is $D > 0$ rather than 1, while the participation cost k remains fixed. From the contest structure, equilibrium evidence expenditure scales linearly with the prize: each party spends $De^c(\theta)$, where $e^c(\theta)$ is the normalized equilibrium effort from Lemma 1. The expected posterior $\pi^e(\theta)$ is unchanged because it depends on the evidence ratio E_P/E_Q; the expected damage payment is therefore $D\pi^e(\theta)$.

The defendant's total expected cost from trial is $D(\pi^e(\theta) + e^c(\theta)) + k$, while the default-judgment payment is D. Q therefore defends if and only if $\pi^e(\theta)+e^c(\theta)+k/D \leq 1$, and P sues if $\pi^e(\theta) - e^c(\theta) \geq k/D$. As stakes grow, the fixed participation cost becomes negligible relative to the prize, so both conditions are easier to satisfy and the trial regime expands. Furthermore, in the extended game with settlement stage, the equilibrium settlement payment is

$$S^*(\theta; D) = \min\{D,\ D(\pi^e(\theta) + e^c(\theta)) + k\}. \tag{23}$$

Following the social-loss criterion in (22), the market-impact wedge $\Lambda\, S^*(\theta; D)$ is increasing in D: higher stakes amplify the downstream distortion from enforcement threats.

To see the effect of presumption in this setting with prize scaling, fix (m, α) and the presumption pair $(\theta_{PV}, \theta_{NP}) = (\alpha + (1-\alpha)m,\ m)$, and suppose the $\min\{\cdot\}$ constraint in (23) is not binding for either prior. Then the presumption-induced shift in the settlement transfer is

$$S^*(\theta_{PV}; D) - S^*(\theta_{NP}; D) = D[(\pi^e(\theta_{PV}) + e^c(\theta_{PV})) - (\pi^e(\theta_{NP}) + e^c(\theta_{NP}))] \geq 0,$$

because the fixed participation costs cancel and $\pi^e(\theta)+e^c(\theta)$ is increasing in θ. The shift is therefore positive and homogeneous of degree one in D: the magnitude of the presumption's effect on extraction scales one-for-one with stakes. Consequently, in high-stakes environments, the presumption amplifies both the magnitude of settlement transfers and the market distortions identified in Section 5.2. This connects the model to concerns about weak patents asserted in high-stakes licensing and litigation environments (Lemley and Shapiro, 2005).

5.4 Institutional variations and robustness

The baseline framework is deliberately stylized to obtain closed-form equilibrium characterizations. This subsection discusses how the core mechanism relates to two practically relevant departures: presumptions that vary with examination or screening history, and environments in which evidence production is asymmetric.

State-dependent presumptions. In practice, the strength of the presumption of validity is not uniform across patents and proceedings. For example, in the U.S. the presumption applies in district-court litigation, whereas no presumption (or a weakened one) applies in post-issuance administrative proceedings before the USPTO's PTAB, such as inter partes review and post-grant review (Rai and Vishnubhakat, 2019; Helmers and Love, 2023; see the institutional discussion in the Introduction). A natural extension is to allow the presumption to depend on observable examination or challenge history.

To capture such state dependence parsimoniously, let z denote an observable screening or examination-history signal (e.g., whether a patent has survived a post-grant challenge) and let the presumption weight vary with z, so that the court's prior under the presumption is

$$\theta(z) = \alpha(z) + (1 - \alpha(z))\, m,$$

where m is the patent's objective merit as in the baseline and $\alpha(z) \in [0, 1]$ indexes the strength of the presumption given z. This formulation highlights a design implication that follows directly from our core mechanism. Recall from our analyses in Sections 4.2–4.3 that the presumption saves resources at trial partly by dampening evidence incentives, but it can increase adjudication error precisely in high-uncertainty environments in which additional information has high marginal value. If higher z corresponds to more informative prior screening on average, then allowing $\alpha(z)$ to be higher for well-vetted patents can save litigation resources with a smaller loss in accuracy (by the logic of Propositions 3 and 4), whereas keeping $\alpha(z)$ lower when screening is weak helps preserve incentives to generate information when it is most valuable. Conditioning the presumption on examination history can therefore mitigate the expenditure–accuracy tension identified in the baseline model.

Asymmetric evidence production. A natural question is how our results extend to environments in which evidence production is asymmetric. In practice, the patent holder often enters litigation with a strong and litigation-ready prosecution record, while the defendant bears the principal burden of assembling invalidity evidence through discovery and prior-art search (Bar and Kalinowski, 2019).

To capture this in the most parsimonious way while keeping the paper's notation and decision rule unchanged, consider a variant of the trial subgame in which P has a fixed evidence baseline $\bar{E}_P > 0$ (for example, a litigation-ready record from prosecution), while only the defendant chooses a resource level $e_Q \geq 0$ that shifts the distribution of E_Q as in (3)–(4). The court's posterior remains (2) with $(E_P, E_Q) = (\bar{E}_P, E_Q)$, so Q's marginal return to spending arises from lowering the posterior in favor of validity. Let $\pi^a(e_Q, \theta)$ denote the resulting expected posterior and let $e^a_Q(\theta)$ denote the defendant's equilibrium evidence-spending level. Because $\bar{E}_P$ is a fixed evidence baseline (its cost is sunk before litigation), P's expected trial-stage payoff is $V^{a,T}_P(\theta) = \pi^a(e^a_Q(\theta), \theta) - k$.

We impose the simplifying condition $\bar{E}_P > he^a_Q(\theta)$. This requires that the plaintiff's prosecution record is sufficiently strong relative to the defendant's cost structure so that even the defendant's highest evidence realization does not overturn the plaintiff's evidentiary advantage. The assumption ensures that the marginal benefit from evidence gathering is monotone in θ, delivering clean comparative statics; it is most naturally satisfied when the plaintiff enters litigation with a strong prosecution record, consistent with the asymmetric-evidence environment motivating this variant. The following proposition documents the comparative statics, which are established for priors $\theta \in [1/2, 1)$. The monotonicity argument used in the proof (included in Supplementary Appendix B) requires this restriction because the marginal benefit of the defendant's evidence spending need not be monotone in θ for $\theta < 1/2$.

Proposition 6. *Consider the asymmetric-evidence trial variant described above. Assume* *that at any interior equilibrium, we have $\bar{E}_P > he^a_Q(\theta)$. Then, for priors $\theta \in [1/2, 1)$:*

(i) *Any interior equilibrium level $e^a_Q(\theta)$ is weakly decreasing in θ.*

(ii) *The expected posterior $\pi^a(e^a_Q(\theta), \theta)$ and the threat-point magnitude $T^a(\theta) := \pi^a(e^a_Q(\theta), \theta) + e^a_Q(\theta) + k$ are weakly increasing in θ.*

The proposition confirms that the core channel identified in the baseline, *i.e.*, the presumption dampening evidence incentives and raising the patent holder's threat point, survives when evidence production is one-sided. The take-it-or-leave-it settlement logic of subsection 5.1 carries over with T^a replacing T: because $T^a(\theta)$ is weakly increasing in θ, a stronger presumption weakly expands settlement leverage in this variant as well.

6 Conclusion

Trial expenditures are not purely wasteful in our framework. Although legal spending does not create new goods, the resources devoted to evidence generate information that affects the court's posterior belief and the accuracy of patent-rights allocation. An error of judgment is costly to society, just as resource dissipation is (Buzzacchi and Scellato, 2008). Our efficiency analysis therefore has two components: the direct resource cost of dispute resolution and the indirect cost of erroneous adjudication.

In this context, the findings of Propositions 3 and 4 reflect the potential trade-off associated with the presumption criterion when there is sufficient uncertainty about the patent's merit. In particular, for patents with merit m between the two thresholds, m^R_{PV} and m^E_{PV}, placing on either side of 1/2, the presumption has contrasting effects. In this case, introducing the presumption will likely raise the error of judgment, but decrease the resource dissipation. This is because, for m close to 1/2, the intensity of competition is at its peak. The bias induced by the presumption likely reduces the intensity, because of which fewer resources would be dissipated. However, it would also reduce the incentive to gather new evidence, which would rather be highly needed especially when there is great uncertainty about the patent's merit. Section 5.2 further discusses a reduced-form welfare criterion that augments adjudication accuracy and resource dissipation with a market-impact wedge, capturing how enforcement threats can tax business activity and technology adoption. Because the presumption raises the equilibrium payment obligation, the desirability of a stronger presumption depends not only on the relative social costs of false positives and false negatives but also on the intensity of enforcement-driven distortions in the relevant market environment.

This finding suggests that we should pay careful attention to the application of the presumption, especially in contexts where the examination of patent applications is complex and invalid patents are granted more frequently by the patent office (e.g., high technology sectors). Examining patent applications is becoming increasingly difficult for several reasons, including the growing number of applications and the budgetary constraints the patent office faces. Granting of invalid patents is an ever-growing reality and the possibility of resolving disputes through legal proceedings is an inevitable consequence (Buzzacchi and Scellato, 2008; Farrell and Shapiro, 2008; de Rassenfosse et al., 2021). Accordingly, in these contexts where m is close to $1/2$, our findings support the arguments against the application of the presumption, which should not be accepted, or even dismissed *tout court*.

Several directions for future work remain open. For instance, fee shifting, as under the English rule, alters the payoff structure by making the losing party bear both sides' litigation costs. Qualitatively, we conjecture that this heightened stake in the outcome would increase the marginal return to evidence production for both parties, since each party's expenditure now affects not only the probability of winning but also whether it bears the opponent's costs. Thus, even when the presumption biases the prior in favor of the patent holder, the additional cost exposure from losing may sustain stronger evidence-seeking incentives for both parties, potentially making the expenditure–accuracy trade-off less severe than under the American Rule assumed in our baseline. However, as noted in footnote 16, fee shifting can also compromise the existence of pure-strategy equilibria in the litigation contest (Massenot et al., 2021), making a formal analysis substantially more involved.

Bifurcation raises a structurally different set of issues. Our baseline model as an invalidity defense within a single infringement proceeding reflects a non-bifurcated setup, as in the UK or Italy, where asserting invalidity is a common defense strategy (Cremers et al., 2016). In bifurcated systems, such as those in Germany and China, infringement and validity are adjudicated in separate proceedings, potentially before different decision-makers, under different evidentiary rules, and with different presumption standards (Cremers et al., 2016). This institutional separation has several implications for our framework. First, it alters the game form: the invalidity challenge becomes a distinct strategic interaction with its own continuation values and threat-point structure, rather than a defense embedded within the infringement trial. Second, it changes the timing of when validity-related evidence costs are sunk relative to settlement bargaining at the infringement stage, thereby modifying the threat point even if the core channel (the presumption biasing the prior and shifting evidence incentives) remains operative. Third, bifurcation can produce divergent outcomes: a patent may initially be deemed infringed in one proceeding but subsequently invalidated in another (Cremers et al., 2016), creating strategic interdependencies between the two proceedings that a single-stage model does not capture. The U.S. system presents a hybrid structure: infringement and invalidity are addressed simultaneously in district courts, but the inter partes review process before the USPTO's PTAB adjudicates validity separately under a lower standard of proof (Rai and Vishnubhakat, 2019; Helmers and Love, 2023). This institutional variation further underscores how bifurcation, in its various forms, would require substantial modifications to the baseline framework.

Our settlement extension in subsection 5.1 assumes complete information, under which settlement absorbs trial on the equilibrium path. Introducing informational frictions, such as private information about patent merit, could restore equilibrium trials as a costly-signaling device, and would allow the model to connect to the empirical regularity that some disputes proceed to trial despite the availability of settlement. A related extension concerns settlement timing: in our framework, settlement occurs before evidence expenditure is sunk; in practice, parties often settle only after incurring discovery-related costs, which would change which expenditures enter the threat point and could alter the presumption's net effect on equilibrium transfers.

In legal practice, the presumption of validity is often discussed jointly with the burden and standard of proof (Lichtman and Lemley, 2007). Our reduced-form representation treats the presumption as a factor that biases the decision-maker's prior belief before evidence is observed, while evidence production and the probabilistic decision rule determine how the posterior responds to realized evidence. This abstraction isolates how a pro-validity starting point alters the marginal return to evidence and therefore litigation incentives. Formally modeling alternative institutional arrangements, such as changing the burden of proof while holding priors fixed, would require modifying the mapping from evidence into adjudication probabilities, which can affect equilibrium characterization and tractability. We leave such extensions to future work, noting that the central comparative-statics channel identified here, through which procedural rules affect behavior via continuation values and evidence incentives, is likely to remain operative.

Appendix A

Appendix A contains the proofs that are omitted in the main text. We will begin with documentation of two additional results, Lemmas A.1 and A.2, that will be useful in proving our main findings. Proofs of these additional lemmas and the proof of Proposition 6 are provided in Supplementary Appendix B.

Lemma A.1. *The equilibrium expenditure, e^c, is increasing in $\theta \in [0, 1/(h^\mu + 1)]$ and decreasing in $\theta \in [h^\mu/(h^\mu + 1), 1]$. Further, if $h^\mu \leq 2$, then e^c is concave in $\theta \in [0, 1]$.*

Lemma A.2. *For $\theta \in [0, 1]$ and $\gamma \in [-1, 1]$, define $F(\theta, \gamma) := \pi^e(\theta) + \gamma e^c(\theta)$. Then, $F(\theta, \gamma)$ is strictly increasing in θ for a given γ.*

Proof of Lemma 1. At the trial stage, the participation cost is sunk, and therefore it does not affect the optimal choice of resource spending. The first-order condition of maximizing U_P with respect to e_P gives

$$\frac{q_{h1}\theta\left(1-\theta\right)\mu h\left(he_P\right)^{\mu-1}\left(e_Q\right)^{\mu}}{\left(\left(1-\theta\right)\left(e_Q\right)^{\mu}+\theta\left(he_P\right)^{\mu}\right)^2}+\frac{q_{1h}\theta\left(1-\theta\right)\mu\left(e_P\right)^{\mu-1}\left(he_Q\right)^{\mu}}{\left(\left(1-\theta\right)\left(he_Q\right)^{\mu}+\theta\left(e_P\right)^{\mu}\right)^2} + \frac{q_0\theta\left(1-\theta\right)\mu\left(e_P\right)^{\mu-1}\left(e_Q\right)^{\mu}}{\left(\left(1-\theta\right)\left(e_Q\right)^{\mu}+\theta\left(e_P\right)^{\mu}\right)^2}=1. \quad \text{(A.1)}$$

Similarly, the first-order condition of maximizing U_Q with respect to e_Q gives

$$\frac{q_{h1}\theta\left(1-\theta\right)\mu\left(he_P\right)^{\mu}\left(e_Q\right)^{\mu-1}}{\left(\left(1-\theta\right)\left(e_Q\right)^{\mu}+\theta\left(he_P\right)^{\mu}\right)^2}+\frac{q_{1h}\theta\left(1-\theta\right)\mu h\left(e_P\right)^{\mu}\left(he_Q\right)^{\mu-1}}{\left(\left(1-\theta\right)\left(he_Q\right)^{\mu}+\theta\left(e_P\right)^{\mu}\right)^2} + \frac{q_0\theta\left(1-\theta\right)\mu\left(e_P\right)^{\mu}\left(e_Q\right)^{\mu-1}}{\left(\left(1-\theta\right)\left(e_Q\right)^{\mu}+\theta\left(e_P\right)^{\mu}\right)^2}=1. \quad \text{(A.2)}$$

There is a symmetric solution $e_P = e_Q = e^c$, satisfying the two first-order conditions simultaneously, such that

$$\frac{q_{h1}\theta\left(1-\theta\right)\mu h^{\mu}}{\left(\left(1-\theta\right)+\theta h^{\mu}\right)^2}+\frac{q_{1h}\theta\left(1-\theta\right)\mu h^{\mu}}{\left(\left(1-\theta\right)h^{\mu}+\theta\right)^2}+q_0\theta\left(1-\theta\right)\mu=e^c. \quad \text{(A.3)}$$

After simplifying, (A.3) reduces to $e^c = \theta\left(1-\theta\right)\mu\mathit{\Gamma}\left(m,\gamma,h,\mu,\theta\right)$, where $\mathit{\Gamma}\left(m,\gamma,h,\mu,\theta\right)$ is given by (10). It can be shown that the solution of the first-order condition also satisfies the second-order condition when $\mu \leq 1$. To see this, note that P's expected payoff, U_P, is a linear combination of terms of the form $xe_P^{\mu}/\left(ye_Q^{\mu}+xe_P^{\mu}\right)$ for different expressions of $x, y > 0$, with positive coefficients, and $(-e_P)$. The second-order derivative of U_P will, therefore, be a linear combination of terms of the following forms with positive coefficients:

$$\frac{d^2}{de_P^2}\frac{xe_P^{\mu}}{\left(ye_Q^{\mu}+xe_P^{\mu}\right)}=\frac{xy\mu e_P^{2\mu-2}e_Q^{\mu}\left[y\left(\mu-1\right)-x\left(\mu+1\right)\right]}{\left(ye_Q^{\mu}+xe_P\right)^3},$$

which is strictly negative for any $x, y > 0$ if $e_p > 0$, $e_Q > 0$, and $\mu \leq 1$. Therefore, U_P is globally concave for $e_p > 0$, $e_Q > 0$.

Similarly, Q's expected payoff, U_Q, is a linear combination of terms of the form $xe_P^{\mu}/\left(ye_Q^{\mu}+xe_P^{\mu}\right)$ for different expressions of $x, y > 0$, with negative coefficients, and $(-e_Q)$. Therefore, the second-order derivative of U_Q will be a linear combination of terms of the following forms with negative coefficients:

$$\frac{d^2}{de_Q^2}\frac{xe_P^{\mu}}{\left(ye_Q^{\mu}+xe_P^{\mu}\right)}=\frac{xy\mu e_P^{\mu}e_Q^{2\mu-2}\left[y\left(\mu+1\right)+x\left(1-\mu\right)\right]}{\left(ye_Q^{\mu}+xe_P\right)^3},$$

which is strictly positive for any $x, y > 0$ if $e_p > 0$, $e_Q > 0$, and $\mu \leq 1$. Because of the negative coefficients, the second-order derivative of U_Q is, therefore, negative and so U_Q

is globally concave for $e_p > 0$, $e_Q > 0$. Further, the first-order conditions (A.1) and (A.2) being strictly positive at $(e_P = 0, e_Q = e^c)$ and $(e_P = e^c, e_Q = 0)$, respectively, implying that the solution e^c is indeed a global maxima for each player, given the other player plays e^c. □

Proof of Lemma 2. It follows from Lemma A.2 that $F(\theta, -1) - k = \pi^e(\theta) - e^c(\theta) - k$ is strictly increasing in θ; and the continuity of F implies that we can find a threshold $\underline{\theta}$, which is increasing in k, such that $F(\theta, -1) - k \geq 0$ if and only if $\theta \geq \underline{\theta}$. Further, $F(0, -1) - k = -k$, and therefore, $\underline{\theta} = 0$ if $k = 0$. Similarly, Lemma A.2 finds that $F(\theta, 1) - (1 - k) = \pi^e(\theta) - (1 - e^c(\theta) - k)$ is increasing and its continuity with respect to θ implies the existence of a threshold $\overline{\theta}$ such that $F(\theta, 1) - (1 - k) \leq 0$ if and only if $\theta \leq \overline{\theta}$. Further, $F(1, 1) - (1 - k) = k$, and therefore, $\overline{\theta} = 1$ if $k = 0$.

□

Proof of Lemma 3. Because $R = 2k$ at $\theta = 0, 1$, and because of the concavity of e^c by Assumption 1, $R > 2k$ for all $\theta \in (0, 1)$ and $\theta^R \in (0, 1)$ uniquely solves $dR/d\theta = 0$, which is equivalent to $de^c/d\theta = 0$ and can be expressed as (17).

Further, because of the concavity of e^c, $\theta^R < 1/2$ if the left-hand-side of (17) is strictly negative at $\theta = 1/2$, and $\theta^R > 1/2$ if the left-hand-side of 17 is strictly positive at $\theta = 1/2$. The left-hand-side of (17), when computed at $\theta = 1/2$, is given by

$$\begin{aligned}
&\frac{8q_{h1}h^{\mu}\left(h^{\mu}+1\right)}{\left(h^{\mu}+1\right)^3}\left(\frac{1}{h^{\mu}+1}-\frac{1}{2}\right)+\frac{8q_{1h}h^{\mu}\left(h^{\mu}+1\right)}{\left(h^{\mu}+1\right)^3}\left(\frac{h^{\mu}}{h^{\mu}+1}-\frac{1}{2}\right)\\
=&\frac{4q_{h1}h^{\mu}\left(1-h^{\mu}\right)}{\left(h^{\mu}+1\right)^3}+\frac{8q_{1h}h^{\mu}\left(h^{\mu}-1\right)}{\left(h^{\mu}+1\right)^3}\\
=&-\frac{4h^{\mu}\left(h^{\mu}-1\right)}{\left(h^{\mu}+1\right)^3}\left[q_{h1}-q_{1h}\right]\\
=&-\frac{16h^{\mu}\left(h^{\mu}-1\right)}{\left(h^{\mu}+1\right)^3}\left(\gamma-\frac{1}{2}\right)\left(m-\frac{1}{2}\right),
\end{aligned}$$

which is strictly negative if $m > 1/2$, and is strictly positive if $m < 1/2$. Therefore, $\theta^R < 1/2$ if $m > 1/2$, and $\theta^R > 1/2$ if $m < 1/2$. Finally, if $m = 1/2$, then $\theta = 1/2$ is a solution of (17), implying $\theta^R = 1/2$. □

Proof of Proposition 3. We let θ_{PV} and θ_{NP} denote the priors under the presumption of validity and the presumption of no validity, respectively. Therefore, $\theta_{PV} = \alpha + (1 - \alpha)\, m$ and $\theta_{NP} = m$. We let R_{PV} and R_{NP} denote the value of R under the presumption of validity and the presumption of no validity, respectively. Specifically, $R_{PV} = R(\theta_{PV})$ and $R_{NP} = R(\theta_{NP})$. Note that $R(0) = R(1) = 2k$, and by Assumption 1, R is concave in θ with a maximum at θ^R.

First, consider $m \geq 1/2$. Then, by Lemma 3, $\theta^R \leq 1/2$ and by concavity, θ^R is decreasing in $\theta \in \left[\theta^R, 1\right]$. Further, as $\theta_{PV} > \theta_{NP} = m \geq \theta^R$, we have $R(\theta_{PV}) < R(\theta_{NP})$. Hence, for all $m \geq 1/2$, $R_{PV} < R_{NP}$.

Next, we consider $m < 1/2$. Then, $\theta^R > 1/2$ and therefore, $m < \theta^R$. As $R(\theta)$ is concave with a unique maximum at θ^R, it follows that for every $\theta < \theta^R$, there exists some $f(\theta) \in \left[\theta^R, 1\right]$ such that $R(\theta) = R(f(\theta))$, $R(\theta) < R(\theta')$ for all $\theta' \in (\theta, f(\theta))$ and $R(\theta) > R(\theta')$ for all $\theta' \in (f(\theta), 1]$. Further, observe that the mapping $f(\theta)$ is decreasing in $\theta < \theta^R$, which follows from the fact that $R(\theta)$ is decreasing in the range $\left[\theta^R, 1\right]$.

Now, consider some $m < 1/2$ such that $R_{NP}(m) = R(\theta_{NP}(m)) < R(\theta_{PV}(m)) = R_{PV}(m)$, which is equivalent to $m < \theta_{PV}(m) < f(m)$ by construction of f. Then, for all $m' < m$, we have $\theta_{PV}(m') < \theta_{PV}(m)$ because θ_{PV} is increasing in m, and $f(m') > f(m)$ because f is decreasing in $m < \theta^R$. Together, it follows that $\theta_{PV}(m') < \theta_{PV}(m) < f(m) < f(m')$ and consequently, $R_{NP}(m') = R(\theta_{NP}(m')) < R(\theta_{PV}(m')) = R_{PV}(m')$. Next, consider some $m < 1/2$ such that $R_{NP}(m) > R_{PV}(m)$, which is equivalent to $f(m) < \theta_{PV}(m)$ by construction of f. Then, for all $m' \in (m, 1/2)$, we have $\theta_{PV}(m) < \theta_{PV}(m')$ because θ_{PV} is increasing in m, and $f(m') < f(m)$ because f is decreasing in $m < \theta^R$. Together, it follows that $f(m') < f(m) < \theta_{PV}(m) < \theta_{PV}(m')$ and consequently, $R_{PV}(m') = R(\theta_{PV}(m')) < R(\theta_{NP}(m')) = R_{NP}(m')$.

We have just shown that for any $m < 1/2$, if $R_{NP}(m) < R_{PV}(m)$, then for all $m' < m$, $R_{NP}(m') < R_{PV}(m')$, and if $R_{PV}(m) < R_{NP}(m)$, then for all $m' > m$, $R_{PV}(m') < R_{NP}(m')$. This observation, together with the fact $R_{PV}|_{m=1/2} < R_{NP}|_{m=1/2}$, implies that $R_{NP} < R_{PV}$ if and only if m is less than some threshold and the threshold is less than 1/2. We refer to this threshold by m^R_{PV}. □

Proof of Lemma 4. To prove part (i), consider the derivative of E with respect to θ:

$$\frac{dE(\theta)}{d\theta} = \frac{q_1 h^\mu}{((1-\theta) + \theta h^\mu)^2} + q_2 + \frac{q_3 h^\mu}{((1-\theta) h^\mu + \theta)^2},$$

where

$$\begin{aligned} q_1 &= (1-m)(1-\gamma)^2 - m\gamma^2 = (1-\gamma)^2 - m\left[(1-\gamma)^2 + \gamma^2\right], \\ q_2 &= 2(1-2m)\gamma(1-\gamma), \\ q_3 &= (1-m)\gamma^2 - m(1-\gamma)^2 = \gamma^2 - m\left[(1-\gamma)^2 + \gamma^2\right]. \end{aligned}$$

Observe that q_1, q_2, and q_3 are linear and decreasing in m. Further, as $1-\gamma < \gamma$, when $m < 1/2$, $q_2 > 0$, $q_3 > 0$, and for sufficiently low values of m, $q_1 > 0$. It follows that there is a threshold value $\underline{m} < 1/2$ such that for any $m \leq \underline{m}$, all q_1, q_2, and q_3 are positive and $dE(\theta)/d\theta > 0$ for all $\theta \in [0,1]$. Similarly, when $m > 1/2$, $q_2 < 0$, $q_1 < 0$, and for sufficiently high values of m, $q_3 < 0$. Therefore, there is a threshold value $\overline{m} > 1/2$ such that for any $m \geq \overline{m}$, all q_1, q_2, and q_3 are negative and $dE(\theta)/d\theta < 0$ for all $\theta \in [0,1]$. This completes the proof of part (i) of the lemma.

To prove part (ii), observe that $q_1 < q_3$ and

$$h^\mu / \left((1-\theta) + \theta h^\mu\right)^2 \lesseqgtr h^\mu / \left((1-\theta) h^\mu + \theta\right)^2 \Leftrightarrow \frac{1}{2} \lesseqgtr \theta.$$

Let us first consider $m \leq 1/2$ and $\theta \geq 1/2$. From $m \leq 1/2$, it follows that $q_2 \geq 0$ and $q_3 > 0$. If $q_1 \geq 0$, then it trivially follows that $dE(\theta)/d\theta \geq 0$. Consider the possibility that $q_1 < 0$. Further, $m \leq 1/2$ implies that $-q_3 < q_1$, or, equivalently, $q_1 + q_3 > 0$. From $\theta \geq 1/2$, it follows that $h^\mu / \left((1-\theta) + \theta h^\mu\right)^2 \leq h^\mu / \left((1-\theta) h^\mu + \theta\right)^2$. Applying these observations, we get

$$dE(\theta)/d\theta = \frac{q_1 h^\mu}{\left((1-\theta) + \theta h^\mu\right)^2} + q_2 + \frac{q_3 h^\mu}{\left((1-\theta) h^\mu + \theta\right)^2} \geq \frac{h^\mu (q_1 + q_3)}{\left((1-\theta) + \theta h^\mu\right)^2} + q_2 \geq 0.$$

Next, consider $m \geq 1/2$ and $\theta \leq 1/2$. From $m \geq 1/2$, it follows that $q_2 \leq 0$, $q_1 < 0$. If $q_3 \leq 0$, it trivially follows that $dE^{LT}(\theta)/d\theta \leq 0$. Consider the possibility that $q_3 > 0$. Further, $m \geq 1/2$ implies that $q_1 < -q_3$, or, equivalently, $q_1 + q_3 < 0$. From $\theta \leq 1/2$, it follows that $h^\mu / \left((1-\theta) + \theta h^\mu\right)^2 \geq h^\mu / \left((1-\theta) h^\mu + \theta\right)^2$. Applying these observations, we get q$_2$

$$dE(\theta)/d\theta = \frac{q_1 h^\mu}{\left((1-\theta) + \theta h^\mu\right)^2} + q_2 + \frac{q_3 h^\mu}{\left((1-\theta) h^\mu + \theta\right)^2} \leq \frac{h^\mu (q_1 + q_3)}{\left((1-\theta) + \theta h^\mu\right)^2} + q_2 \leq 0.$$

When $m = 1/2$, the analysis above suggests that $E(\theta)$ is increasing in θ for $\theta \geq 1/2$ and is decreasing in θ for $\theta \leq 1/2$, implying that $E(\theta)$ reaches its minimum at $\theta = 1/2$. This completes the proof of part (ii) of the lemma. □

Proof of Proposition 4. We let θ_{PV} and θ_{NP} denote the priors under the presumption of validity and the presumption of no validity, respectively. Therefore, $\theta_{PV} = \alpha + (1-\alpha) m$ and $\theta_{NP} = m$. Similarly, we let E_{PV} and E_{NP} denote the value of E under the presumption of validity and the presumption of no validity, respectively. Specifically, $E_{PV} = E(\theta_{PV})$ and $E_{NP} = E(\theta_{NP})$. To prove the part (i) of the proposition, we will show that $\Delta_{PV} := E_{PV} - E_{NP} \geq 0$ for all $m \leq 1/2$.

Consider the range of parameter values such that the litigation trial regime prevails in equilibrium in both the presumption scenarios, PV and NP. Therefore, using (20), $\Delta_{PV} = E(\theta_{PV}) - E(m)$ can be expressed as

$$\Delta_{PV} = h^\mu \alpha (1-m) \left[\frac{q_1}{ab} + \frac{q_3}{cd} + \frac{q_2}{h^\mu}\right],$$

where

$$q_1 = (1-\gamma)^2 - m\left(\gamma^2 + (1-\gamma)^2\right),\ q_2 = 2\gamma(1-\gamma)(1-2m),$$
$$q_3 = \gamma^2 - m\left(\gamma^2 + (1-\gamma)^2\right),\ a = (1-\theta_{PV}) + \theta_{PV} h^{\mu},$$
$$b = (1-m) + m h^{\mu},\ c = (1-m) h^{\mu} + m,\ d = (1-\theta_{PV}) h^{\mu} + \theta_{PV}.$$

Observe that for $m \leq 1/2$, $q_2 \geq 0$ and $q_3 > 0$.

For $m \leq (1-\gamma)^2 / \left(\gamma^2 + (1-\gamma)^2\right) < 1/2$, we have $q_1 \geq 0$, and it therefore trivially follows that $\Delta_{PV} \geq 0$.

For $m \in \left((1-\gamma)^2 / \left(\gamma^2 + (1-\gamma)^2\right), 1/2\right]$, we have $q_1 < 0$, $q_2 \geq 0$, and $q_3 > 0$. In this range of values of m, $(q_1/ab) + (q_3/cd) \geq 0$ implies $\Delta_{PV} \geq 0$.

Claim 1: $(q_1/ab) + (q_3/cd) \geq 0$ at $m = 1/2$.

Proof of Claim 1: At $m = 1/2$, we have $a > b = (h^{\mu} + 1)/2 = c > d$ and $q_3 = -q_1 = \gamma^2 - (1-\gamma)^2$. Therefore,

$$(q_1/ab) + (q_3/cd) = \frac{2\left(\gamma^2 - (1-\gamma)^2\right)}{h^{\mu} + 1}\left(\frac{1}{d} - \frac{1}{a}\right) > 0,$$

which proves Claim 1.

Claim 2: (q_1/ab) is decreasing in m for $m \in \left((1-\gamma)^2 / \left(\gamma^2 + (1-\gamma)^2\right), 1/2\right]$.

Proof of Claim 2: Consider the derivative of (q_1/ab) (for brevity, we write df/dm as f' when f is a function of m):

$$\frac{d}{dm}\frac{q_1}{ab} = \frac{(q_1)' ab - q_1 (ab)'}{(ab)^2} \leq 0 \Leftrightarrow (q_1)' ab \leq q_1 (ab)' \Leftrightarrow \frac{(q_1)'}{q_1} \geq \frac{(ab)'}{ab}. \tag{A.4}$$

The inequality is reversed in the last part of the above chain of equivalent conditions because $q_1 < 0$. Note that

$$\frac{(q_1)'}{q_1} = \frac{\left(\gamma^2 + (1-\gamma)^2\right)}{m\left(\gamma^2 + (1-\gamma)^2\right) - (1-\gamma)^2} > 0$$

for $m \in \left((1-\gamma)^2 / \left(\gamma^2 + (1-\gamma)^2\right), 1/2\right]$ and is decreasing in m. Therefore, its minimum value is reached at $m = 1/2$ and

$$\min_{m \in \left(\frac{(1-\gamma)^2}{\gamma^2 + (1-\gamma)^2}, 1/2\right]} \frac{(q_1)'}{q_1} = \frac{2\left(\gamma^2 + (1-\gamma)^2\right)}{\gamma^2 - (1-\gamma)^2} > 2. \tag{A.5}$$

Further,

$$\frac{(ab)'}{ab} = \frac{a'}{a} + \frac{b'}{b} = \frac{(1-\alpha)(h^{\mu} - 1)}{(1-\theta_{PV}) + \theta_{PV} h^{\mu}} + \frac{(h^{\mu} - 1)}{(1-m) + m h^{\mu}},$$

and each of the two terms is less than 1 by Assumption 1. Therefore,

$$\max_{m\in\left(\frac{(1-\gamma)^2}{\gamma^2+(1-\gamma)^2},1/2\right]}\frac{(ab)'}{ab}\leq 2. \tag{A.6}$$

Therefore, (A.4) must hold for $m\in\left((1-\gamma)^2/\left(\gamma^2+(1-\gamma)^2\right),1/2\right]$, and therefore, (q_1/ab) is decreasing in this range of values of m, which completes the proof of Claim 2.

Claim 3: (q_3/cd) is decreasing in m for $m\in\left((1-\gamma)^2/\left(\gamma^2+(1-\gamma)^2\right),1/2\right]$.

Proof of Claim 3: Consider the derivative of (q_3/cd):

$$\begin{aligned}\frac{d}{dm}\frac{q_3}{cd}&=\frac{(q_3)'\,cd-q_3\,(cd)'}{(cd)^2}\leq 0\\ \Leftrightarrow (q_3)'\,cd\leq q_3\,(cd)'\Leftrightarrow\frac{(q_3)'}{q_3}&\leq\frac{(cd)'}{cd}=\frac{c'}{c}+\frac{d'}{d}.\end{aligned} \tag{A.7}$$

Observe that

$$\begin{aligned}\frac{(q_3)'}{q_3}&=-\frac{\left(\gamma^2+(1-\gamma)^2\right)}{\gamma^2-m\left(\gamma^2+(1-\gamma)^2\right)},\\ \frac{c'}{c}&=-\frac{(h^\mu-1)}{h^\mu-m\,(h^\mu-1)},\\ \frac{d'}{d}&=-\frac{(1-\alpha)\,(h^\mu-1)}{h^\mu-\theta_{PV}\,(h^\mu-1)},\end{aligned}$$

and, therefore, (A.7) can be rewritten as

$$\frac{(h^\mu-1)}{h^\mu-m\,(h^\mu-1)}+\frac{(1-\alpha)\,(h^\mu-1)}{h^\mu-\theta_{PV}\,(h^\mu-1)}\leq\frac{\left(\gamma^2+(1-\gamma)^2\right)}{\gamma^2-m\left(\gamma^2+(1-\gamma)^2\right)}. \tag{A.8}$$

Further, note that

$$\begin{aligned}\frac{(1-\alpha)\,(h^\mu-1)}{h^\mu-\theta_{PV}\,(h^\mu-1)}&\leq\frac{(h^\mu-1)}{h^\mu-m\,(h^\mu-1)}\\ \Leftrightarrow (1-\alpha)\,h^\mu-(1-\alpha)\,m\,(h^\mu-1)&\leq h^\mu-\theta_{PV}\,(h^\mu-1)\\ \Leftrightarrow \alpha\,(h^\mu-1)&\leq\alpha h^\mu,\end{aligned}$$

which holds true. Therefore, to prove (A.8), it is sufficient to show that

$$\frac{2\,(h^\mu-1)}{h^\mu-m\,(h^\mu-1)}\leq\frac{\left(\gamma^2+(1-\gamma)^2\right)}{\gamma^2-m\left(\gamma^2+(1-\gamma)^2\right)}.$$

We can simplify the above inequality as follows:

$$\begin{aligned}
&\frac{2\left(h^{\mu}-1\right)}{h^{\mu}-m\left(h^{\mu}-1\right)} \leq \frac{\left(\gamma^{2}+(1-\gamma)^{2}\right)}{\gamma^{2}-m\left(\gamma^{2}+(1-\gamma)^{2}\right)} \\
\Leftrightarrow & 2\left(h^{\mu}-1\right)(1-m) \gamma^{2}-2\left(h^{\mu}-1\right) m(1-\gamma)^{2} \\
& \leq \gamma^{2}\left(h^{\mu}-m\left(h^{\mu}-1\right)\right)+(1-\gamma)^{2}\left(h^{\mu}-m\left(h^{\mu}-1\right)\right) \\
\Leftrightarrow & -(1-\gamma)^{2}\left(2\left(h^{\mu}-1\right) m+h^{\mu}-m\left(h^{\mu}-1\right)\right) \\
& \leq \gamma^{2}\left(h^{\mu}-m\left(h^{\mu}-1\right)-2\left(h^{\mu}-1\right)(1-m)\right) \\
\Leftrightarrow & -(1-\gamma)^{2}\left(\left(h^{\mu}-1\right) m+h^{\mu}\right) \leq \gamma^{2}\left(2-h^{\mu}+m\left(h^{\mu}-1\right)\right),
\end{aligned}$$

which holds true, because the right-hand side is positive by Assumption 1 and the left-hand side is negative. This proves Claim 3.

Claims 1, 2, and 3 show that $(q_1/ab)+(q_3/cd) \geq 0$ at $m = 1/2$ and $(q_1/ab)+(q_3/cd)$ is decreasing in m for $m \in \left((1-\gamma)^2/\left(\gamma^2+(1-\gamma)^2\right), 1/2\right]$. Therefore,$(q_1/ab)+(q_3/cd) \geq 0$, and consequently, $\Delta_{PV} \geq 0$ for $m \in \left((1-\gamma)^2/\left(\gamma^2+(1-\gamma)^2\right), 1/2\right]$. As we have already shown that $\Delta_{PV} \geq 0$ for $m \leq (1-\gamma)^2/\left(\gamma^2+(1-\gamma)^2\right)$, it follows that $\Delta_{PV} \geq 0$ for $m \leq 1/2$.

At $m = 1$, $\theta_{PV} = m$, and $\Delta_{PV} = E^{LT}(\theta_{PV}) - E^{LT}(m) = 0$. Δ_{PV} can be negative for $m > 1/2$; however, because of continuity, we can find a threshold $m^E_{PV} \geq 1/2$ such that $\Delta_{PV} \geq 0$ if $m \leq m^E_{PV}$. Further, note that the result provides only a sufficient condition, but not a necessary condition, because Δ_{PV} is not monotone in m. □

Proof of Proposition 5. Q accepts an offer S if and only if $-S \geq V^T_Q(\theta)$, or equivalently $S \leq -V^T_Q(\theta) = \pi^e(\theta) + e^c(\theta) + k = T(\theta)$. Because P has full bargaining power, she chooses the largest acceptable transfer subject to $S \in [0,1]$, which yields $S^*(\theta) = \min\{1, T(\theta)\}$. The settlement surplus relative to trial is

$$[S+(-S)]-\left[V_P^T(\theta)+V_Q^T(\theta)\right]=2\left(e^c(\theta)+k\right) \geq 0,$$

which is strictly positive whenever $e^c(\theta)+k > 0$; therefore, settlement weakly dominates trial in the continuation game. Further, if Q defaults she obtains -1, whereas if she defends her equilibrium payoff is $-S^*(\theta) \geq -1$, with strict inequality if $T(\theta) < 1$. Thus Q weakly prefers defending to default judgment. Also, if P does not sue she obtains 0, whereas if she sues her equilibrium payoff is either 1 (under default judgment) or $S^*(\theta) > 0$ for $\theta \in (0,1)$. Hence P strictly prefers suing to not suing. Finally, since Lemma A.2 implies that $T(\theta)$ is strictly increasing in θ, and $\theta_{PV} = \alpha + (1-\alpha)m > \theta_{NP} = m$, it follows that $T(\theta_{PV}) > T(\theta_{NP})$. Applying the increasing map $x \mapsto \min\{1,x\}$ yields $S^*(\theta_{PV}) \geq S^*(\theta_{NP})$, with strict inequality whenever $T(\theta_{NP}) < 1$. □

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