Supplementary materials for "Presumption of patent validity and litigation incentives"

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Appendix B

This document contains supplemental materials for Guerra and Kundu, "Presumption of patent validity and litigation incentives." It has two sections. In Section B.1, we present proofs of Lemma A.1, Lemma A.2, and Proposition 6. In Section B.2, we discuss some key assumptions in our model.

B.1 Additional proofs

Proof of Lemma A.1. Observe that

$$\frac{de^c}{d\theta} = \mu\Bigg[q_{h1}h^\mu\frac{d}{d\theta}\frac{\theta(1-\theta)}{((1-\theta)+\theta h^\mu)^2} + q_0\frac{d\theta(1-\theta)}{d\theta} + q_{1h}h^\mu\frac{d}{d\theta}\frac{\theta(1-\theta)}{((1-\theta)h^\mu+\theta)^2}\Bigg],$$

which, after simplifying, reduces to

$$\frac{de^c}{d\theta} = \mu\Bigg[\frac{q_{h1}h^\mu(h^\mu+1)}{((1-\theta)+\theta h^\mu)^3}\left(\frac{1}{h^\mu+1}-\theta\right) + 2q_0\left(\frac{1}{2}-\theta\right) + \frac{q_{1h}h^\mu(h^\mu+1)}{((1-\theta)h^\mu+\theta)^3}\left(\frac{h^\mu}{h^\mu+1}-\theta\right)\Bigg].$$

Because $0 < 1/(h^\mu+1) < 1/2 < h^\mu/(h^\mu+1) < 1$, $(de^c/d\theta)$ is strictly positive for all $\theta \leq 1/(h^\mu+1)$ and $(de^c/d\theta)$ is strictly negative for all $\theta \geq h^\mu/(h^\mu+1)$, these observations together prove the first part of the lemma. Further, it follows that the global maximum lies in $[1/(h^\mu+1), h^\mu/(h^\mu+1)]$ and multiple local optima might exist for $\theta \in [1/(h^\mu+1), h^\mu/(h^\mu+1)]$, depending on the curvature of e^c. To study the curvature, we examine

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the second-order derivative.

$$\frac{d^2e^c}{d\theta^2} = \mu\Bigg[q_{h1}h^\mu \frac{d}{d\theta}\frac{1-\theta\,(h^\mu+1)}{((1-\theta)+\theta h^\mu)^3} + q_0\frac{d\,(1-2\theta)}{d\theta} + q_{1h}h^\mu \frac{d}{d\theta}\frac{h^\mu-\theta\,(h^\mu+1)}{((1-\theta)\,h^\mu+\theta)^3}\Bigg].$$

which, after simplifying, reduces to

$$\frac{d^2e^c}{d\theta^2} = \mu\Bigg[\frac{2q_{h1}h^\mu\,(h^{2\mu}-1)}{((1-\theta)+\theta h^\mu)^4}\left(\theta-\frac{2h^\mu-1}{h^{2\mu}-1}\right) - 2q_0 + \frac{2q_{1h}h^\mu\,(h^{2\mu}-1)}{((1-\theta)\,h^\mu+\theta)^4}\left(\frac{h^\mu\,(h^\mu-2)}{h^{2\mu}-1}-\theta\right)\Bigg].$$

Observe that if $h^\mu \leq 2$, then $(2h^\mu-1)/(h^{2\mu}-1) \geq 1$ and $h^\mu(h^\mu-2)/(h^{2\mu}-1) \leq 0$, which together imply $(d^2e^c/d\theta^2)$ is negative, or equivalently, e^c is globally concave for $\theta \in [0,1]$. □

Proof of Lemma A.2. Observe that

$$\begin{aligned}
F(\theta) =& \pi^e(\theta)+\gamma e^c(\theta)\\
=& q_{h1}\left(\frac{\theta h^\mu}{(1-\theta)+\theta h^\mu}+\frac{\theta\,(1-\theta)\,\gamma\mu h^\mu}{((1-\theta)+\theta h^\mu)^2}\right)+q_0\,(\theta+\theta\,(1-\theta)\,\gamma\mu)\\
&+q_{1h}\left(\frac{\theta}{(1-\theta)\,h^\mu+\theta}+\frac{\theta\,(1-\theta)\,\gamma\mu h^\mu}{((1-\theta)\,h^\mu+\theta)^2}\right)\\
=& \frac{q_{h1}h^\mu\theta\,(h^\mu\theta+(1+\gamma\mu)\,(1-\theta))}{((1-\theta)+\theta h^\mu)^2}+q_0\,(\theta+\theta\,(1-\theta)\,\gamma\mu)\\
&+\frac{q_{1h}\theta\,(\theta+h^\mu\,(1+\gamma\mu)\,(1-\theta))}{((1-\theta)\,h^\mu+\theta)^2}.
\end{aligned}$$

The first-order derivatives of the three components of F are as follows:

$$\begin{aligned}
(i)\;& \frac{d}{d\theta}\frac{q_{h1}h^\mu\theta\,(h^\mu\theta+(1+\gamma\mu)\,(1-\theta))}{((1-\theta)+\theta h^\mu)^2}\\
&= \frac{q_{h1}h^\mu}{((1-\theta)+\theta h^\mu)^3}\Bigg[(1-\theta)(1+\gamma\mu)+\theta h^\mu(1-\gamma\mu)\Bigg];\\
(ii)\;& \frac{d}{d\theta}q_0\,(\theta+\theta\,(1-\theta)\,\gamma\mu) = q_0\,(1+\gamma\mu(1-2\theta))\,;\\
(iii)\;& \frac{d}{d\theta}\frac{q_{1h}\theta\,(\theta+h^\mu\,(1+\gamma\mu)\,(1-\theta))}{((1-\theta)\,h^\mu+\theta)^2}\\
&= \frac{q_{1h}h^\mu}{((1-\theta)\,h^\mu+\theta)^3}\Bigg[h^\mu\,(1+\gamma\mu)\,(1-\theta)+(1-\gamma\mu)\,\theta\Bigg];
\end{aligned}$$

Each of these derivatives is strictly positive for $\theta \in (0,1)$ when $0 < \mu \leq 1$, since $1+\gamma\mu \geq$

$1-\mu \geq 0$ and $1-\gamma\mu \geq 1-\mu \geq 0$ for any $\gamma \in [-1,1]$. Therefore, $F(\theta,\gamma)$ is strictly increasing in $\theta \in [0,1]$. □

Proof of Proposition 6. Consider the asymmetric-evidence variant in which P's evidence is fixed at $\bar{E}_P > 0$ and only Q chooses $e_Q \geq 0$. The court's posterior remains

$$\pi(\bar{E}_P, E_Q, \theta) = \frac{\theta(\bar{E}_P)^\mu}{(1-\theta)(E_Q)^\mu + \theta(\bar{E}_P)^\mu},$$

which is strictly decreasing in E_Q for any $\theta \in (0,1)$ and $\mu > 0$. Given e_Q, the random evidence E_Q takes values e_Q and he_Q, with $\Pr(E_Q = e_Q \mid s = V) = \gamma$ and $\Pr(E_Q = e_Q \mid s = I) = 1-\gamma$ (with $\gamma > 1/2$). Taking expectations with respect to the presumption-free belief about s (probability m of V) yields two ex-ante weights on the events $E_Q = e_Q$ and $E_Q = he_Q$:

$$\tilde{q}_1 := \Pr(E_Q = e_Q) = m\gamma+(1-m)(1-\gamma), \qquad \tilde{q}_h := \Pr(E_Q = he_Q) = m(1-\gamma)+(1-m)\gamma,$$

with $\tilde{q}_1 + \tilde{q}_h = 1$.

Define the expected posterior under the asymmetric-evidence technology by

$$\pi^a(e_Q,\theta) := \tilde{q}_1\, \pi(\bar{E}_P, e_Q, \theta) + \tilde{q}_h\, \pi(\bar{E}_P, he_Q, \theta).$$

The defendant's expected payoff in the trial subgame can be written as

$$U_Q^a(e_Q,\theta) = -\pi^a(e_Q,\theta) - e_Q - k,$$

so any interior optimum $e_Q^a(\theta) > 0$ satisfies the first-order condition

$$-\frac{\partial \pi^a(e_Q,\theta)}{\partial e_Q} = 1. \tag{B.1}$$

Because $\pi(\bar{E}_P, E_Q, \theta)$ is decreasing in E_Q and E_Q scales linearly in e_Q in both events, the left-hand side of (B.1) is strictly positive for interior solutions. Moreover, for $0 < \mu \leq 1$ each component $\pi(\bar{E}_P, x, \theta)$ is convex in $x > 0$ (the second derivative with respect to x is positive), so π^a is convex in e_Q and $U_Q^a = -\pi^a - e_Q - k$ is strictly concave in e_Q; the FOC (B.1) is therefore both necessary and sufficient for an interior maximum.

With these preliminaries in place, we prove the two parts of the proposition in turn.

Next, fix $x > 0$ and define the single-event marginal benefit of Q's spending (up to the positive weights $\tilde{q}_1, \tilde{q}_h$ and the factors $\partial x/\partial e_Q \in \{1, h\}$) as

$$M_x(\theta, e_Q) := \frac{\theta(1-\theta)(\bar{E}_P)^\mu \mu\, x^{\mu-1}}{\big((1-\theta)x^\mu + \theta(\bar{E}_P)^\mu\big)^2},$$

so that $-\partial\pi^a/\partial e_Q$ is a positive linear combination of M_{e_Q} and M_{he_Q} with fixed weights given by $(\tilde{q}_1, \tilde{q}_h)$ and $(1, h)$. Write $a := x^\mu$ and $b := (\bar{E}_P)^\mu$, and consider $f_x(\theta) := \theta(1-\theta)/\left(a+\theta(b-a)\right)^2$ so that M_x is proportional to $f_x(\theta)$ for fixed (x, e_Q). Differentiating,

$$f_x'(\theta) = \frac{(a+\theta(b-a))\Big((1-2\theta)(a+\theta(b-a)) - 2\theta(1-\theta)(b-a)\Big)}{\left(a+\theta(b-a)\right)^4}.$$

The bracket simplifies to $a(1-\theta)-\theta b$, which is strictly negative for all $\theta \in [1/2, 1)$ whenever $b > a$, i.e. whenever $(\bar{E}_P)^\mu > x^\mu$. Under the condition $\bar{E}_P > he_Q$ we have $(\bar{E}_P)^\mu > e_Q^\mu$ and $(\bar{E}_P)^\mu > (he_Q)^\mu$, so for each $x \in \{e_Q, he_Q\}$ the mapping $\theta \mapsto M_x(\theta, e_Q)$ is strictly decreasing on $\theta \in [1/2, 1)$ for every fixed $e_Q > 0$. Hence $\theta \mapsto -\partial\pi^a(e_Q, \theta)/\partial e_Q$ is strictly decreasing on $[1/2, 1)$ for every fixed $e_Q > 0$. At an interior optimum satisfying (B.1), a higher θ therefore strictly shifts down the marginal benefit schedule while the marginal cost of e_Q remains equal to one; by the strict concavity established above, any interior solution $e_Q^a(\theta)$ is weakly decreasing in θ on $[1/2, 1)$ (with strict decrease whenever e_Q^a remains interior).

Finally, the expected posterior $\pi^a(e_Q^a(\theta), \theta)$ responds to a higher θ through two reinforcing channels: (i) a *direct* effect, since $\pi(\bar{E}_P, E_Q, \theta)$ is increasing in θ for fixed E_Q; and (ii) an *indirect* effect, since a weak reduction in equilibrium e_Q implies a weak reduction in E_Q in both events (because $E_Q \in \{e_Q, he_Q\}$), and $\pi(\bar{E}_P, E_Q, \theta)$ is decreasing in E_Q.

It remains to show that the threat-point magnitude $T^a(\theta) := \pi^a(e_Q^a(\theta), \theta) + e_Q^a(\theta) + k$ is weakly increasing in θ. Note that $T^a(\theta) = -U_Q^a(e_Q^a(\theta), \theta)$. By the envelope theorem, at any interior optimum the FOC gives $\partial U_Q^a/\partial e_Q = 0$, so

$$\frac{dT^a}{d\theta} = -\frac{dU_Q^a}{d\theta} = -\frac{\partial U_Q^a}{\partial\theta}\bigg|_{e_Q = e_Q^a} = \frac{\partial\pi^a(e_Q, \theta)}{\partial\theta}\bigg|_{e_Q = e_Q^a} > 0,$$

where the strict inequality follows because $\partial\pi(\bar{E}_P, x, \theta)/\partial\theta = (\bar{E}_P)^\mu x^\mu / \Big((1-\theta)x^\mu + \theta(\bar{E}_P)^\mu\Big)^2 > 0$ for each realization $x \in \{e_Q, he_Q\}$. Together, a higher presumption-induced prior θ weakly increases the defendant's exposure at the threat point and therefore strengthens the presumption's leverage-through-threat-points channel. □

B.2 Discussion of the modeling approach and assumptions

In this section, we discuss our modeling approach and some key assumptions.

Probabilistic decision rule: We consider a probabilistic decision rule to determine the outcome of a litigation contest. An alternative approach is to utilize a deterministic decision rule, similar to those employed in all-pay auction models within the contest literature. For example, P wins if the difference or ratio between the evidence presented by the two parties exceeds a certain threshold.[1] However, this intuitive form of decision rule does come with its drawbacks. The equilibrium of the game typically only exists in mixed strategies. Working with mixed-strategy equilibria is—or at least, appears to us as less appealing in our research, for two reasons. First, in addition to the uncertainty surrounding the true state, there are two additional sources of randomness: one from the equilibrium strategy and the other from the stochastic evidence-production function. These two sources will interact to produce evidence in equilibrium. This interaction complicates the process of updating beliefs based on the observed evidence profile, making the subsequent analysis quite intractable.

Further, the interpretation of a mixed strategy equilibrium poses particular challenges in the context of litigation. A party adheres to a mixed strategy in equilibrium only if it receives the same payoff from every pure action over which it chooses to randomize. In our case, this implies, for example, that the plaintiff is indifferent among a set of expenses she would incur. However, if the plaintiff, instead of playing stochastically, consistently opts for a specific cost level based on her preferences, she would still achieve the same return but would impose an externality on the other player, ultimately disrupting the equilibrium consensus. The consensus relies on the understanding that it is preferable to introduce randomness since no pure action is strictly preferred.[2] In practice, it is challenging to envision how legal firms, which invest substantial time and resources in carefully evaluating every aspect of a case, would agree to embrace a mixed strategy concerning expenses in the first place.

Modeling the presumption bias: While legal scholarship has always regarded the presumption as a source of asymmetry, it remains unclear how this asymmetry is imposed. We approach the asymmetry in terms of a biased belief, where the judge evaluates evidence under the influence of this biased belief. Alternatively, one could consider the possibility that the judge begins with an unbiased belief but examines the evidence in a biased manner. Our motivation for adopting the former approach stems from the observation that pre-litigation activities such as the patent granting procedure may influence the judge to rule in favor of the patent holder. As Moore (2002) highlights, "practitioners and scholars alike have frequently opined that juries are not likely to invalidate patents because juries favor inventors and are unlikely to second-guess the Patent Office that has technically trained examiners who already issued the patents." Thus, the bias may be imposed even before the examination of evidence, and the parties' decisions on resource spending are influenced by this bias within the due process.

In our model, the bias appears in the following form: bias $= (\theta - m) = \alpha\,(1 - m)$. The bias is decreasing in the underlying merit of the patent—a high-merit patent would be

[1] See Konrad (2002), Kirkegaard (2012) for examples of all-pay auction contests, and Konrad (2009) for a discussion of the literature.

[2] This line of critique is also acknowledged in the literature; see Tullock (1985).

less affected by the presumption criterion. An alternative modeling assumption could be considering a constant bias at every level of a patent's merit. However, given the natural bounds of probabilities, the assumption of constant bias is less appealing. Furthermore, we expect the presumption to have a positive influence on the court's belief in favor of patent validity, going beyond the true merit of the patent. In simpler terms, under PV, $\theta - m$ should be positive. Expressing the bias as a specific function of α restricts the generalizability of our analysis. Nonetheless, we have used α in an ad hoc manner: it is necessary to derive a closed-form solution of the litigation contest. However, our analysis of the effect of PV does not explicitly consider derivatives with respect to α.

Interpretation of evidence: In the model, our interpretation of evidence focuses on the intensity of evidence rather than its specific contents. By doing so, we abstract away several crucial aspects of evidence collection, such as the strategies employed by legal teams in pursuing different lines of inquiry and the strategic considerations involved in choosing one aspect over another. While analyzing these concepts can offer additional insights into the strategic decisions related to evidence collection, we maintain the tractability of our framework by representing intensity as a single parameter in our model.

Sensitivity to the evidence: In our model, the parameter $0 < \mu \leq 1$ measures sensitivity to the evidence. We impose an upper bound on to ensure existence of the Nash equilibrium. Alternative interpretations of the effects of μcan be found in Hirshleifer (1989) and Jia (2008). Following Hirshleifer (1989), μ can be interpreted as an index of the mass effect in the contest. It can be shown that the quasi-concavity of the CSF imposes an upper bound on the value of μ. Jia (2008) interprets this parameter as a measure of noise. As μ approaches zero, evidence would have little influence, and the posterior would be similar to the prior. Conversely, as μ approaches infinity, the contest outcome is almost determined by an all-pay auction, and the party with a higher intensity of evidence is guaranteed to win. Nevertheless, for large values of μ, the characterization of equilibrium in contest with non-anonymous CSF remains an unsolved issue.

Assumption 1: This assumption ensures concavity of the resource spending level, which allows us to study comparative static effects on resource dissipation and judgment error in a tractable form. The assumption is not necessary for characterizing the equilibrium regimes or for determining the effect of shifting the prior on the equilibrium regime. Relaxing this assumption would impact the uniqueness of θ^R. If the assumption is violated, e^c is still increasing for $\theta \leq 1/(1 + h^\mu(\mu - 1))$ and is decreasing for $\theta \geq h^\mu/(h^\mu + \mu - 1)$; however, there could be multiple local maxima for the intermediate values of θ. Therefore, in the absence of Assumption 1, we will still observe that the presumption is going to increase (decrease) the aggregate resource dissipation for sufficiently low-merit (high-merit) patents; however, the uniqueness of the threshold on the θ-values will not hold.

Additional references cited in the supplementary appendix

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