Presumption of patent validity and litigation incentives

Alice Guerra[*]   Tapas Kundu[†]

December 16, 2024

Abstract

We analyze the effects of the presumption of patent validity on litigation incentives. We consider a litigation game between a patent holder and an alleged infringing firm. A court resolves the dispute if there is a trial. We model the court's decision-making as a learning process based on evidence and consider the presumption as a factor influencing the court's prior belief of patent validity. The presumption affects the trial outcome in two ways—by biasing the prior, and by affecting the incentives to invest in evidence-seeking activities. We find that the presumption can affect the incidence of trials in either direction. We further reveal its countervailing efficiency effects—likely raising the error of judgment, but decreasing the resource dissipation—when there is high uncertainty about the patent's objective merit. In these contexts, our findings suggest a cautious or limited application of the presumption.

JEL classification: C72, D74, D83, K11, K41, O34

Keywords: patents, litigation, presumption, learning, persuasion, error of judgment

---

[*]Affiliation: Department of Economics, University of Bologna.
E-mail: alice.guerra3@unibo.it. Address: via Angherà 22, 47921 Rimini, Italy. Phone: +39 0541 434 266. ORCID: 0000-0003-1956-0270.

[†]Corresponding author. Affiliation: Oslo Business School, Oslo Metropolitan University.
Email: tapas.kundu@oslomet.no. Address: Pilestredet 35, 0166 Oslo, Norway. Phone: +47 67 23 50 47. ORCID: 0000-0003-1814-7230.

1 Introduction

Granted patents generally enjoy a presumption of validity in court proceedings (Seaman, 2019). The burden of proof falls upon the infringer, who has to provide relevant facts about patent invalidity and convince the court with clear and convincing evidence, which is a higher standard of proof than the "preponderance of evidence" typically required in civil lawsuits.[1] In common law countries, by statute "a patent shall be presumed valid" and the "burden of establishing invalidity of a patent or any claim thereof shall rest on the party asserting such invalidity" (35 USC ∮282, 1952).[2] A similar presumption holds for European patents and Supplementary Protection Certificates (Graham et al., 2002; Seaman, 2019).

There are different theoretical justifications in support of the presumption of [patent] validity—the most common one being deference to the patent office's expertise in evaluating patent applications, along with its agency flexibility and political accountability (Devlin, 2008; Seaman, 2019).[3] Notwithstanding this general rationale, the application of the presumption is not uniform neither across jurisdictions, nor across legal proceedings within a jurisdiction (Ottoz, 2019). Differences across national jurisdictions are substantial, especially within Europe (Graham et al., 2002; Graham and Van Zeebroeck, 2013; Cremers et al. 2017). For example, the Netherlands apply no presumption, whereas Sweden and Denmark generally apply a strong presumption of validity unless there is clear evidence of invalidity. Recent case laws have also considered a reversed presumption, namely a presumption of invalidity (e.g., *Syral Belgium v. Roquette Frères).*[4]

---

[1]To clarify some of the terminology that will be used in our analysis, the standard of proof is the level of certainty and the amount of evidence necessary to prove a claim in a trial; the burden of proof identifies the party who must offer evidence to raise a claim in litigation (Sanchirico, 1997). These concepts are practically interdependent but theoretically distinct (Clermont and Sherwin, 2002; Schwartz and Seaman, 2013; Guerra et al., 2019; 2022).

[2]The Congress amended Section 282 in 1965, 1975, 1995, and 2011. The sentences: "A patent shall be presumed valid," and "The burden of establishing invalidity of a patent or any claim thereof shall rest on the party asserting such invalidity," remained in the statute. See, e.g., *Microsoft Corp. v. i4i Limited Partnership*, 131 S.Ct. 2238 (2011).

[3]See also *Cuozzo Speed Techs. v. Lee*, 136 S. Ct. 2131, 2144 (2016).

[4]*Syral Belgium v. Roquette Frères* (Supreme Court, Belgium, 12 September 2014, Case No. C.13.0232) challenged the *prima facie* validity of a patent based on a decision from other European jurisdictions (UK and France) invalidating the patent. In this case, a foreign legal decision may generate a presumption of invalidity for the Belgian part of the patent.

Differences across legal proceedings within a jurisdiction are substantial as well. In the U.S., the presumption of validity is applied in court litigation, whereas no presumption or a "weakened" presumption is applied in post-issuance administrative proceedings before the U.S. Patent and Trademark Office's (USPTO) Patent Trial and Appeal Board (PTAB). These proceedings—through which the public may ask the patent office to re-assess the validity of granted patents—include inter partes review, post-grant review, and ex parte re-examination (Rai and Vishnubhakat, 2019; Helmers and Love, 2023).[5] For example, in the inter partes review—which has been introduced by the America Invents Act on September 16, 2012—a petitioner can challenge the validity of a U.S. patent under a "preponderance of evidence" standard, showing that claims are more likely unpatentable than not (Helmers and Love, 2023).[6] This significantly reduces the standard of proof, compared to patent litigation in court where the "clear and convincing evidence" standard is applied. Consequently, an infringement defendant may prefer to challenge patent validity in post-issuance administrative proceedings, rather than in or in parallel with litigation in district courts (Rai and Vishnubhakat, 2019; Seaman, 2019; Helmers and Love, 2023).

Beyond the discrepancies in the application of the presumption of validity across jurisdictions and legal proceedings, several critiques have been raised upon the mere presence of the presumption. Some scholars argued that the presumption should be abolished because it is not simply a procedural device, but rather a powerful mechanism for injecting pro-patentee bias (Bohrer, 2004; Bock, 2014). An explicit statement in jury instructions that a patent is presumed valid makes juries less likely to invalidate patents (Moore, 2002). This is not problematic if the patent was correctly granted: in this

---

[5]Regarding inter partes review, "In an inter partes review [...], the petitioner shall have the burden of proving a proposition of unpatentability by a preponderance of the evidence." 35 U.S.C. ∮ 316(e). See also *In re* Global Tel*link Corp., IPR 2014-00493, 2014 WL 4715524, Sept. 17, 2014 ("There is no presumption of validity as to the challenged claims in an inter partes review."). Regarding ex parte re-examination, "the standard of proof – a preponderance of evidence – is substantially lower than in a civil case [and] there is no presumption of validity." (*In re* Swanson, 540 F.3d 1368, 1377, Fed. Cir. 2008).

[6]35 U.S.C. § 316(e) (inter partes review), 326(e) (post-grant review). See also the Trial Practice and Procedure Rules, confirming that "the default evidentiary standard is a preponderance of the evidence" (37 CFR § 42.1(d)). For more details about the inter partes review process, see, e.g., Helmers and Love (2023).

case, the presumption protects the patent holder from any frivolous (or, non-meritorious) infringement litigation (Seaman, 2019). The problem arises when patent offices make evaluation errors and grant patents that, on their merits, should not have been issued in the first place (de Rassenfosse et al., 2021).[7] These patents are referred to in the literature as "latently invalid" or "incorrectly granted" patents (Henkel and Zischka, 2019), and also as "bad" or "weak" patents (Choi and Gerlach, 2015; Lei and Wright, 2017).

The problem of latently invalid patents is sizable, especially in the U.S. (Commission, 2004; Lemley and Shapiro, 2005; The Economist, 2015).[8] Judicial review is generally one solution to correct latently invalid patents. Yet, only a small fraction of all patents is litigated—a mere 0.1%, as noted by Lemley and Shapiro (2005)— because of both the expensive district court litigation process and the presence of the presumption of validity (American Intellectual Property Law Association, 2013; Helmers and Love, 2023).

Rather than entering litigation, innovators may find it more convenient to pay licensing and transaction costs of bargaining to reach a private agreement with the patentee, thereby leaving the latently invalid patent in the market (Kesan and Gallo, 2006). This exponentially increases the uncertainties of the innovation process by exposing innovators to the risk of non-meritorious litigation (Farrell and Shapiro, 2008; de Rassenfosse et al., 2021). A major driver of non-meritorious patent infringement litigation is represented by non-practicing entities (NPE; also called "patent trolls")—that is, patent holders whose sole purpose is to enforce patent rights to threaten litigation and demand licensing fees, rather than producing or selling products or services (Pénin, 2012; Ganglmair et al., 2022).[9] The presumption of validity may have the double-edged effect of

---

[7]Patent evaluation errors by patent offices as the USPTO may occur for different reasons, including low accuracy levels of reviewing patent applications, or inefficient incentives to grant valid patents (e.g., absence of penalties for incorrectly issued patents). Evaluation errors are particularly common in high technology sectors, which are not areas of traditional patenting, and therefore it is more difficult for the patent office to gather prior art information (Kesan and Gallo, 2006). On the risk of errors by both patent examiners at patent offices and judges at civil court, see Buzzacchi and Scellato (2008).

[8]The problem is also present in Europe, but less pronounced (Cremers et al., 2017; Henkel and Zischka, 2019). For example, Cremers et al. (2017) reported that about 30% of appealed patent suits have their initial decision overturned; Henkel and Zischka (2019) found a 75% invalidity rate of appeals at the German Federal Patent Court between 2000 and 2012. See also Palangkaraya et al. (2011), which analyzed patent applications granted by the USPTO and examined at both the European Patent Office and Japanese Patent Office during the 1990s. Their estimates reveal that 9.8% of patents were incorrectly granted.

[9]A classic example of a legal case involving a patent troll is *NTP, inc. v. Research in Motion*, 397 F.

leveraging NPEs' litigation tactics and their abilities to extract licensing payments from producing firms (Patent Quality Improvement Hearings, 2003, p. 4).

The debate upon the presumption has not been limited to scholarly contributions. During court litigation cases, judges and juries discussed the application of the presumption and the level of validity which should be applied to issued patents (for a review, see Klimczak, 2012). For example, the Supreme Court articulated and confirmed the application of the presumption at common law in several legal cases—one of the most cited being *Microsoft Corp. v. i4i Ltd. Partnership*.[10] In other legal cases, the Supreme Court stated that the presumption can be weakened or even eliminated when the patent office did not or was not able to consider relevant prior art in its review of patent applications.[11]

The legal debates on the presumption, along with the discrepancies in its application across jurisdictions and legal proceedings, circle back to a basic, yet fundamental question: What are the effects of the presumption on litigation incentives? To our knowledge, no prior studies thus far formally addressed this question. This gap in the literature is problematic, especially considering that legal and empirical studies have suggested the important role of legal presumptions in patent litigation (Chatlynne, 2010; Schwartz and Seaman, 2013; Bock, 2014; Henkel and Zischka, 2019; Seaman, 2019),[12] and some scholars and governmental institutions have repeatedly advocated reforms to overcome the presumption in court proceedings (Devlin, 2008; Ottoz, 2019; Seaman, 2019).

In this article, we seek to fill this knowledge gap by providing an analytical framework that explains how the legal presumption affects litigation incentives and its efficiency

---

Supp. 2d 785 (E.D. Va. 2005), where a NTP (a Virginia-based patent holding company) sued Research in Motion (RIM, the manufacturers of Blackberry) for patent infringement over its Blackberry devices. Even if the patent office found that three disputed patents should have not been granted in the first place, the NTP won an out-of-court settlement of US$ 612 million from RIM. For a discussion of this and other patent litigation cases involving patent trolls, see, e.g., Chan and Fawcett (2005).

[10] *Microsoft Corp. v. i4i Ltd. Partnership*, 131 S. Ct. 2238, 564 U.S. 91, 180 L. Ed. 2d 131 (2011). For an extensive discussion on this legal case, see Schwartz and Seaman (2013). See also *Radio Corp. of America v. Radio Engineering Laboratories, Inc.*, 293 U.S. 1, 55 S. Ct. 928, 79 L. Ed. 163 (1934); *Austin Machinery Co. v. Buckeye Traction Ditcher Co.*, 13 F.2d 697 (6th Cir. 1926); *Coffin v. Ogden*, 85 U.S. 120, 21 L. Ed. 821 (1874); *Sciele Pharma Inc. v. Lupin Ltd.*, 684 F.3d 1253 (Fed. Cir. 2012).

[11] See, e.g., *Manufacturing Research Corp. v. Graybar Elec. Co.*, 679 F.2d 1355 (11th Cir. 1982); *Heyl & Patterson, Incorporated v. McDowell Company*, 317 F.2d 719 (4th Cir. 1963).

[12] Some theoretical contributions analyzed the effects of legal presumptions on individuals' choices in other contexts (e.g., in tort settings; Bernardo et al., 2000; Demougin and Fluet, 2008; Guerra et al., 2022).

implications. We present a tractable patent-litigation game between a patent holder and a potential infringing firm. A non-strategic decision-maker, for example, a judge or a jury in case of court proceedings, resolves the dispute if there is a trial. The game consists of three sequential stages: the patent holder's decision to sue; the infringing firm's decision to defend; the trial stage where parties invest resources to gather and present evidence in order to persuade the court to make a favorable decision. The court's prior belief of patent validity is influenced by the presumption criterion, whereas the posterior belief takes the evidence produced during the litigation trial into account. The presumption affects the outcome of a trial in two ways—by biasing the prior, and by affecting the incentive to invest in evidence-seeking activities.

We characterize the competing parties' litigation decisions and their investment decisions that arise in equilibrium. We show that the effect of the presumption on the possibility of a litigation trial can be ambiguous. We further analyze its effect on two important features of a litigation trial—resource dissipation and error of judgment. Both resource dissipation and error of judgment are inefficient for the society. One of the key insights from our analysis is that the presumption has countervailing efficiency effects when there is high uncertainty about the patent's objective merit, e.g., in contexts where the examination of patent applications is complex and invalid patents are granted more frequently by the patent office (e.g., in high technology sectors).

The countervailing effects arise because of how the presumption influences the incentive to invest in evidence-seeking activities. Specifically, when the decision-maker is highly uncertain about the patent's validity, the competing parties act aggressively. However, the presumption of validity biases the competition in favor of the patent holder. On the one hand, the bias dampens the intensity of competition, which in turn reduces resource dissipation—and this is, in general, socially beneficial. On the other hand, it also reduces parties' incentives to gather new evidence, which has a high positive learning effect especially when there is high uncertainty about the patent's merit.

Our model of the litigation game is inspired by Skaperdas and Vaidya (2012): we model the litigation-trial stage game as a contest with Bayesian learning, a probabilistic

decision rule, and a stochastic evidence-production function.[13] However, our approach departs from theirs in several ways. First, we introduce the presumption as a factor that influences the decision-maker's prior belief of patent validity, before examining any evidence. This approach helps to study how the tension between adjusting belief due to the presumption and that due to learning from evidence, affects the litigation incentives. Second, we consider a specific stochastic evidence-production function that gives closed-form solutions of the equilibrium strategies, which enable us to study the comparative static effects in a tractable manner. Last but not least, our focus is different from theirs. Skaperdas and Vaidya (2012) intend to provide an information-based foundation of the additive contest success function (CSF) in a litigation contest, whereas we consider the Bayesian inferential process at the trial stage as a building block, and focus on how a biased belief arising from a legal presumption affects litigation incentives.

Our model shares common features with the contest framework used to analyze rent-seeking incentives in various applications in economics and political science, including conflict, lobbying, litigation, advertising, internal and external labor markets, and R&D competition; see, for example, Konrad (2000; 2009) and the references therein. In much of the existing works involving contests with fixed prizes, efforts are non-productive and purely rent-seeking in nature. In our framework, contest efforts can lead to hard information that fosters a proper allocation of property rights. In this sense, we contribute to the informational lobbying literature. Formal models of strategic information provision in lobbying can be found in Austen-Smith and Wright (1992), Bennedsen and Feldmann (2002; 2006), and Lagerlöf (1997; 2007). Related to our work is Lagerlöf (2007), in which multiple firms compete in a rent-seeking contest to win the monopoly right. These firms, based on their private but soft information, make costly investments to acquire hard information to persuade a regulator with the authority of granting a monopoly. Unlike Lagerlöf (2007), we do not consider the possibility of privately informed rent-seekers. Further, we follow a different line of inquiry. Lagerlöf (2007) studies the welfare implication of costly information acquisition, whereas we examine how a biased prior belief affects

---

[13]On modeling contest success functions in litigation contests, see, among many others, Hirshleifer and Osborne (2001); Farmer and Pecorino (1999).

information-seeking incentives and litigation incentives.

Our treatment of the information transmission process also differs from the recent literature on Bayesian persuasion in contests (Zhang and Zhou, 2016; Clark and Kundu, 2021). In these models of Bayesian persuasion, a contest designer strategically commits to a information-disclosure mechanism to rent-seeking agents. In our model, the rent-seeking agents disclose hard information to persuade the uninformed decision-maker. Heterogeneity among players is commonly acknowledged as a limiting factor to investments in contest (Chowdhury et al., 2023). Our model share common features with this literature, particularly in analyzing how the inherent asymmetry due to the presumption bias extends to litigation incentives.

The rest of the article is organized as follows. Section 2 introduces the model setup and assumptions. Section 3 characterizes the equilibrium. Section 4 examines the effects of the presumption on litigation incentives, resource dissipation, and judicial error in the court's decision-making process. Section 5 highlights some key features of our study and concludes with suggestions for future research. The appendix A includes the proofs omitted in the main text, while the online appendix B includes proofs of some additional results used in the main analysis, and a discussion of the model and its key assumptions.

2 Model

2.1 A patent-litigation game

We consider a litigation game between a patent holder P and a potential infringing firm Q, both assumed to be risk neutral. The game proceeds in three stages.

- Stage 1: P decides whether to sue Q. If P does not sue, the game ends and both players receive their default payoffs, which are normalized to zero. Otherwise, the game moves to stage 2.

- Stage 2: Q decides whether to defend. If Q decides not to defend, she submits to a default judgment in which the court awards P damages without holding a trial.

Without loss of generality, we normalize the damage compensation value to 1. If Q defends, the game moves to a trial at stage 3.

- Stage 3: If the game proceeds to the trial stage, then both parties incur a positive participation cost $k \in [0, 1]$. In addition, the competing parties strategically spend costly resources to gather and present evidence favorable to their causes. Specifically, at the beginning of the trial, P and Q simultaneously choose the level of resources $e_P \geq 0$ and $e_Q \geq 0$, respectively. Based on the evidence and the legal environment, which reflects the merit of the patent and the presumption of its validity, the court determines whether the patent is deemed valid. We consider a probabilistic decision-making process, which we discuss in the following subsections. If the court finds the patent valid, Q pays P the damage compensation. Otherwise, she pays nothing. The game ends with the court's decision.

We normalize the expenditure function such that the costs of using resources of level e are also given by e.[14] The model assumes the American Rule regarding the payment of legal expenditures, with each party paying their own legal expenditures.[15]

There are two types of costs associated with a trial: a fixed participation cost and resources spent in collecting evidence. The participation costs involve costs, such as time spent and psychological stress, that parties must endure even if they opt not to allocate resources to gather evidence. In contrast, players strategically allocate their resources to improve their chances to secure a favorable outcome. In a default judgment, parties can avoid paying any trial-related costs.

We assume that the outcome of the lawsuit will not affect the market outcome. This assumption simplifies the payoff structure of the game—in our model, the parties are solely disputing over a transfer and they have equal valuation of the prize (see the Online Appendix for more discussion).

---

[14]A convex cost function is sufficient for the existence of an interior solution to the payoff-maximization problem. The specific functional form is beneficial for deriving a closed-form solution of the Nash equilibrium.

[15]With the English Rule, the litigation game may not have pure-strategy equilibrium for all parameter values. On the American vs English Rule, see, e.g., Massenot et al. (2021).

2.2 Belief about the patent validity

Following Skaperdas and Vaidya (2012), we model the court's decision-making as a process of persuasion. The alleged infringing firm, Q, can defend itself by denying infringement, challenging the validity of the patent, or both. This scenario resembles a non-bifurcated legal system with an invalidity argument as a defense. In non-bifurcated systems, such as those in the UK or Italy, there is no separation between infringement and validity proceedings. Defendants can challenge a patent's validity within infringement proceedings. Conversely, in bifurcated legal systems like those in Germany and China, infringement and validity proceedings are handled separately in distinct courts (Cremers et al., 2016; see the Online Appendix for further discussion).

Our model focuses on an invalidity argument in a patent infringement litigation. Specifically, we develop a persuasion framework that focuses on seeking favorable evidence by both parties in relation to the question of validity. Assume that there are two possible states $s \in \{V, I\}$ of the world: one in which the patent is valid (V), and the other in which the patent is invalid (I). The court's prior, denoted by θ, and posterior, denoted by π, quantify its belief about the event $s = V$, before and after it considers the evidence produced during the trial, respectively.

The court's prior is a subjective assessment of the patent's validity before examining evidence. We assume that a patent's underlying merit and the presumption criterion influence the prior. The merit of a patent refers to the presumption-free belief about the state of patent's validity. We consider two presumption scenarios: one in which there is a presumption of validity (PV), and the other in which there is no presumption of validity (NP). We use a simple reduced-form representation of the prior of the following form:

$$\theta = \begin{cases} \alpha + (1 - \alpha)\, m & \text{under PV} \\ m & \text{under NP} \end{cases}, \tag{1}$$

where $m \in (0, 1)$ represents the merit and $\alpha \in (0, 1)$ measures the relative weight on the presumption of validity. We can interpret the court's weighted prior under PV in

the following way. A patent's validity can be assessed based on multiple criteria. One can find patent-specific free information on some criteria (of proportion $1 - \alpha$) and the patent's intrinsic merit is useful in assessing validity on these grounds. On other criteria (of proportion α), no information is available and the presumption of patent's validity guides the prior assessment. It is worth noting that the effect of the presumption criterion is only limited to influencing the court's prior—it must not alter the competing parties' belief about the underlying state. Therefore, even when there is a presumption of validity, the competing parties' belief must be given by the presumption-free prior, i.e., $s = V$ with probability m.

The updating of the court's belief adheres to Bayes' Rule. Let E_i denote the evidence produced by $i \in \{P, Q\}$ during the trial. Given a prior θ, the court's posterior belief is given by

$$\pi = \Pr\left(s = V \mid E_P, E_Q\right) = \frac{\theta L^V}{(1 - \theta) + \theta L^V},$$

where L^V is the likelihood ratio of patent validity based on evidence and is given by

$$L^V\left(E_P, E_Q\right) = \frac{\Pr\left(E_P, E_Q \mid V\right)}{\Pr\left(E_P, E_Q \mid I\right)}.$$

The likelihood ratio is also a subjective assessment of validity based on evidence. It is common in the literature to consider the likelihood ratio in power-law form (see, e.g., Skaperdas and Vaidya, 2012):

$$L^V\left(E_P, E_Q\right) = \left(\frac{E_P}{E_Q}\right)^\mu,$$

where the pieces of evidence E_P and E_Q are expressed on the $(0, \infty)$ scale. The parameter $0 < \mu \leq 1$ indicates sensitivity to the evidence (see the Online Appendix for more discussion). We can therefore express the posterior probability as

$$\pi\left(E_P, E_Q, \theta\right) = \frac{\theta\left(E_P\right)^\mu}{(1 - \theta)\left(E_Q\right)^\mu + \theta\left(E_P\right)^\mu}. \tag{2}$$

2.3 Production of evidence

We assume that the production of evidence depends on both the resources spent and the underlying state:

$$E_P\left(e_P, s\right) = \begin{cases} he_P & \text{with probability } f\left(s\right) \\ e_P & \text{with probability } 1 - f\left(s\right) \end{cases},$$

$$E_Q\left(e_Q, s\right) = \begin{cases} e_Q & \text{with probability } f\left(s\right) \\ he_Q & \text{with probability } 1 - f\left(s\right) \end{cases}, \tag{3}$$

where $h > 1$, $s \in \{V, I\}$, and

$$f\left(V\right) = 1 - f\left(I\right) = \gamma > \frac{1}{2}. \tag{4}$$

This evidence production function reflects how the true state positively influences the availability of favorable evidence. For example, in state V, in which the patent is valid, P is more likely (with probability $\gamma > 1/2$) to produce a higher volume of favorable evidence compared to Q, given equal resource spending. The opposite effect is observed in state I. We assume that E_P and E_Q are independent variables. The court only observes the pair of evidence (E_P, E_Q) but does not observe the underlying resources P and Q spend.

2.4 A probabilistic decision rule

The court uses the following probabilistic decision rule to arrive at its verdict: Choose V with probability $\pi\left(E_P, E_Q, \theta\right)$ and choose I with probability $(1 - \pi\left(E_P, E_Q, \theta\right))$. The success probabilities of P and Q of winning the trial are therefore given by $\pi\left(E_P, E_Q, \theta\right)$ and $(1 - \pi\left(E_P, E_Q, \theta\right))$, respectively.[16] We let U_P and U_Q denote the expected payoffs of

---

[16]We can express, after some algebraic manipulation, the ratio of winning probabilities as $\pi/(1-\pi) = L^V \cdot \theta/(1-\theta) = (E_P/E_Q)^\mu \cdot \theta/(1-\theta)$. This expression contains elements analogous to the litigation success function derived in Hirshleifer and Osborne (2001), exhibiting certain desirable properties. Following Hirshleifer and Osborne (2001), $\theta/(1-\theta)$ can be interpreted as the "fault factor," which is influenced by the subjective probabilities of the underlying state, at the pre-litigation stage. And, $(E_P/E_Q)^\mu$ can be

P and Q, respectively. We assume that all parameter values are common knowledge and analyze the Bayesian Nash equilibrium of the game.

3 The equilibrium analysis

We begin our analysis at the final stage of the game. Consider the sub-game in which both players arrive at a trial. The participation cost is now sunk and P and Q simultaneously choose their expenditures (e_P, e_Q) to maximize their respective expected payoffs. As the transfer of damage compensation takes place only if P wins, the expected payoffs are as follows:

$$U_P = \mathbb{E}_s\left[\pi\left(E_P, E_Q, \theta\right)\right] - e_P - k,$$

$$U_Q = -\mathbb{E}_s\left[\pi\left(E_P, E_Q, \theta\right)\right] - e_Q - k, \tag{5}$$

where $\mathbb{E}_s[\cdot]$ is the expectation operator over the probability distribution of s.

A profile of resources (e_P, e_Q) can lead to four possible events with distinct profiles of evidence (E_P, E_Q), which are (he_p, e_Q), (he_p, he_Q), (e_p, he_Q), and (e_p, e_Q). We denote these events by $event_{h1}$, $event_{hh}$, $event_{1h}$, and $event_{11}$, respectively, such that the subscript ij refers to the event in which the evidence profile is (ie_P, je_Q), $i \in \{h, 1\}$, $j \in \{h, 1\}$. Because E_P and E_Q are independent, we determine the state-conditional probabilities of these events using the marginal distributions, as described in (3) and (4). Specifically,

$$\Pr\left[event_{h1} \mid s\right] = \begin{cases} \gamma^2 & \text{if } s = V \\ (1-\gamma)^2 & \text{if } s = I \end{cases}; \quad \Pr\left[event_{hh} \mid s\right] = \begin{cases} \gamma(1-\gamma) & \text{if } s = V \\ \gamma(1-\gamma) & \text{if } s = I \end{cases};$$

$$\Pr\left[event_{1h} \mid s\right] = \begin{cases} (1-\gamma)^2 & \text{if } s = V \\ \gamma^2 & \text{if } s = I \end{cases}; \quad \Pr\left[event_{11} \mid s\right] = \begin{cases} \gamma(1-\gamma) & \text{if } s = V \\ \gamma(1-\gamma) & \text{if } s = I \end{cases}. \tag{6}$$

As already discussed, differently from the beliefs about contest success probabilities that

---

interpreted as the "effort factor," which is influenced by the resources spent during the litigation process.

are influenced by court's prior and thus affected by the presumption criterion, the competing parties' beliefs about the state of validity at the time of spending resources are solely guided by the presumption-free prior. We can therefore determine the competing parties' ex-ante belief about $event_{ij}$, denoted by q_{ij}, as

$$q_{ij} := \Pr\left[event_{ij}\right] = \Pr\left[event_{ij} \mid s = V\right]\Pr\left[s = V\right] + \Pr\left[event_{ij} \mid s = I\right]\Pr\left[s = I\right],$$

which gives us

$$q_{h1} = \Pr\left[event_{h1}\right] = m\gamma^2 + (1-m)(1-\gamma)^2,$$

$$q_{hh} = \Pr\left[event_{hh}\right] = \gamma(1-\gamma),$$

$$q_{1h} = \Pr\left[event_{1h}\right] = m(1-\gamma)^2 + (1-m)\gamma^2,$$

$$q_{11} = \Pr\left[event_{11}\right] = \gamma(1-\gamma).$$

For notational convenience, we define $q_0 := q_{hh} + q_{11} = 2\gamma(1-\gamma)$. Then, P's expected payoff given a profile of resources (e_P, e_Q) is given by

$$\begin{aligned}
U_P &= \sum_{i\in\{h,1\},j\in\{h,1\}} \pi(ie_p, je_Q, \theta)\Pr\left[event_{ij}\right] - e_P - k \\
&= \frac{q_{h1}\theta(he_P)^\mu}{(1-\theta)(e_Q)^\mu + \theta(he_P)^\mu} + \frac{q_{1h}\theta(e_P)^\mu}{(1-\theta)(he_Q)^\mu + \theta(e_P)^\mu} \\
&\quad + \frac{q_0\theta(e_P)^\mu}{(1-\theta)(e_Q)^\mu + \theta(e_P)^\mu} - e_P - k
\end{aligned} \tag{7}$$

To derive the terms in the final expression of (7), we apply that $\pi(he_p, he_Q, \theta) = \pi(e_p, e_Q, \theta)$. Similarly, we can write Q's expected payoff as

$$\begin{aligned}
U_Q &= -\sum_{i\in\{h,1\},j\in\{h,1\}} \pi(ie_p, je_Q, \theta)\Pr\left[event_{ij}\right] - e_Q - k \\
&= -\frac{q_{h1}\theta(he_P)^\mu}{(1-\theta)(e_Q)^\mu + \theta(he_P)^\mu} - \frac{q_{1h}\theta(e_P)^\mu}{(1-\theta)(he_Q)^\mu + \theta(e_P)^\mu} \\
&\quad - \frac{q_0\theta(e_P)^\mu}{(1-\theta)(e_Q)^\mu + \theta(e_P)^\mu} - e_Q - k.
\end{aligned} \tag{8}$$

The following lemma characterizes the unique symmetric Nash equilibrium of the sub-game. The proof is reported in the Appendix.

Lemma 1. *In the litigation contest, both parties incur the same expenditure, e^c, which is given by*

$$e^c = \theta \left(1 - \theta\right) \mu \Gamma, \tag{9}$$

where Γ is expressed as a function of m, γ, h, μ, and θ, and is given by

$$\Gamma\left(m, \gamma, h, \mu, \theta\right) = \frac{q_{h1} h^\mu}{\left(\left(1 - \theta\right) + \theta h^\mu\right)^2} + q_0 + \frac{q_{1h} h^\mu}{\left(\left(1 - \theta\right) h^\mu + \theta\right)^2}. \tag{10}$$

In this probabilistic contest, the legal expenditures are rent-seeking and both parties spend the same amount of resources in equilibrium.[17] Nevertheless, there is a possibility of learning from evidence because the court's posterior can differ from its prior. The likelihood ratio of patent validity based on evidence, $L^V\left(E_P, E_Q\right)$, is $\left(he^*/e^*\right)^\mu = h^\mu$ and $\left(e^*/he^*\right)^\mu = 1/h^\mu$ in the events $event_{h1}$ and $event_{1h}$, respectively, and remains at 1 in the events $event_{hh}$ and $event_{11}$. We can describe the transition of the court's belief, from its prior to posterior, in the equilibrium path as follows:

$$\pi\left(E_P, E_Q, \theta\right) = \begin{cases} \frac{\theta h^\mu}{(1-\theta)+\theta h^\mu} & \text{with probability } q_{h1} \\ \theta & \text{with probability } q_0 \\ \frac{\theta}{(1-\theta)h^\mu+\theta} & \text{with probability } q_{1h} \end{cases}. \tag{11}$$

We denote the ex-ante expected value of the posterior by π^e, which is given by

$$\pi^e = \theta \left[\frac{h^\mu q_{h1}}{(1-\theta)+\theta h^\mu} + q_0 + \frac{q_{1h}}{(1-\theta)h^\mu + \theta}\right] \tag{12}$$

---

[17]This is a well-known result from the large rent-seeking literature building on Tullock (1978; 1997), also when applied to litigation (Guerra et al., 2019; Friehe and Wohlschlegel, 2019; Massenot et al., 2021).

The ex-ante expected payoffs of P and Q at the trial are

$$\mathbb{E}\left(U_P\right) = \pi^e - e^c - k,$$

$$\mathbb{E}\left(U_Q\right) = -\pi^e - e^c - k.$$

Next, we analyze stage 2. Consider Q's decision to defend. She can either pay the compensation value of 1 or defend a trial, in which case, her expected payoff is $\mathbb{E}\left(U_Q\right)$. Therefore, Q defends if $-1 \leq \mathbb{E}\left(U_Q\right)$, or, equivalently, if

$$\pi^e \leq 1 - e^c - k. \tag{13}$$

Finally, we analyze stage 1. Consider P's decision to litigate. P receives a zero payoff if she does not litigate. Her payoff from litigation depends on Q's response. In particular, if (13) is not satisfied, then default judgment occurs and P receives a payoff of one. If (13) is satisfied, Q defends a trial, in which case P has a positive expected payoff only if $\mathbb{E}\left(U_P\right) \geq 0$, or equivalently, if $\pi^e \geq e^c + k$. Therefore, P litigates in the following two situations: one in which Q submits to a default judgment, which occurs if $\pi^e \geq 1 - e^c - k$; the other in which Q responds by defending a trial, which occurs if $e^c + k < \pi^e < 1 - e^c - k$. The first situation presents a first-mover advantage that P has in this sequential game. Combining the two conditions, we find that P litigates if

$$\pi^e \geq \min\left\{e^c + k, 1 - e^c - k\right\}. \tag{14}$$

We can now fully characterize the equilibrium outcome based on the relationship between the expected posterior π^e and the cost-based thresholds $(e^c + k)$ and $(1 - e^c - k)$. Three different regimes can arise in equilibrium:

- **Litigation trial**: P litigates and Q defends a trial. This arises if $e^c + k \leq \pi^e \leq 1 - e^c - k$.

- **Default judgment**: P litigates and Q pays the damage compensation. This arises if (13) does not hold.

- **No litigation**: P does not litigate. This arises if (14) does not hold.

The following proposition characterizes the equilibrium outcome of the litigation game. The proof follows straightforwardly from the preceding discussion.

Proposition 1. *The equilibrium regimes of the litigation game are characterized as follows:*

(a) If $\pi^e < \min\{e^c + k, 1 - e^c - k\}$, then the no-litigation regime prevails.

(b) If $\pi^e > 1 - e^c - k$, then the default-judgment regime prevails.

(c) If $e^c + k \leq \pi^e \leq 1 - e^c - k$, then the litigation-trial regime prevails. This range can be vacuous if $e^c + k > 1/2$.

The intuition behind Proposition 1 is straightforward. The expected posterior π^e reflects how players perceive P's chance of winning a trial. Therefore, the incentives of P and Q to engage in a trial are increasing and decreasing in π^e, respectively. The litigation-trial regime prevails, if at all, when the expected posterior falls within the two cost-based threshold values $(e^c + k)$ and $(1 - e^c - k)$.

4 Effects of the presumption of validity

The presumption criterion induces a bias to the prior, which is otherwise determined by the underlying merit of a patent. The merit influences the outcome of the litigation game in two different ways: first, through the prior; and second, by moderating the expected return to investment. The first effect is qualitatively similar to the effect of the bias induced by the presumption of validity—and we are particularly interested to delineate this effect. However, by simply measuring the incremental effect of m, we will also capture the second effect.

To circumvent this challenge, we adopt an indirect approach. We fix the merit m and study how changing a generic prior θ affects various derived parameters of our model. By delineating the effect of a generic prior after controlling for the merit's effect, we get to measure how the presumption-induced bias to the prior would affect the outcome of the

game.[18]

To this end, consider first the effect of a generic prior θ on the expected posterior, π^e, and the resources spent, e^c, for a fixed value of m. It follows from (12) that $d\pi^e/d\theta > 0$, implying that the expected posterior π^e is increasing in θ. The difference between the expected posterior π^e and the prior can be expressed as

$$\pi^e(\theta) - \theta = \theta(1-\theta)(h^\mu - 1)\left[\frac{q_{h1}}{(1-\theta) + \theta h^\mu} - \frac{q_{1h}}{(1-\theta)h^\mu + \theta}\right], \qquad (15)$$

from which it follows that the expected posterior coincides with the prior at $\theta = 0, 1$, and at some $\widehat{\theta}$ that satisfies $q_{h1}/((1-\theta) + \theta h^\mu) = q_{1h}/((1-\theta)h^\mu + \theta)$. Further, whenever $\widehat{\theta} \in (0,1)$, the posterior is above the prior for $\theta \in (0, \widehat{\theta})$ and below the prior for $\theta \in (\widehat{\theta}, 1)$.

The resource-spending level e^c changes non-monotonically with respect to θ; $e^c = 0$ at $\theta = 0, 1$, and $e^c > 0$ for $\theta \in (0,1)$. In Lemma A.1 in the Appendix, we show that the threshold e^c is increasing in $\theta \in [0, 1/(h^\mu + 1)]$ and decreasing in $\theta \in [h^\mu/(h^\mu + 1), 1]$. Further, if $h^\mu \leq 2$, then e^c is concave, and consequently, $1 - e^c$ is convex.

In the following analysis, in the interest of tractability, we assume $h^\mu \leq 2$.[19] This assumption implies that e^c has a unique maximum at some prior $\theta \in [1/(h^\mu + 1), h^\mu/(h^\mu + 1)]$.

Assumption 1. $h^\mu \leq 2$.

4.1 The equilibrium regimes

We first examine how shifting the prior between the two presumption scenarios, PV and NP, affects the equilibrium regimes. Despite the non-monotonic effect of the prior on the resources spent during the trial, we find that the expected trial payoffs of P and Q change monotonically with respect to θ. Consequently, there exist threshold values for the prior delineating players' willingness to engage in a trial. The following lemma documents the

---

[18]Note that we can achieve similar findings by studying the derivatives of π^e and e^c with respect to α while keeping the values of m fixed. However, these derivatives involve complex algebraic expressions, hence becoming less tractable. In contrast, our approach enables us to interpret the findings in terms of the shifting effects of the prior.

[19]In the online Appendix, we discuss how the results would be affected if this assumption is violated.

observation, with the proof reported in the Appendix.

Lemma 2. *For any given $k \geq 0$, there exists thresholds $\underline{\theta} \geq 0$ and $\bar{\theta} \geq 0$ such that $\pi^e \geq e^c + k$ if and only if $\theta \geq \underline{\theta}$, and $\pi^e \leq 1 - e^c - k$ if and only if $\theta \leq \bar{\theta}$. If $k = 0$, $\underline{\theta} = 0$ and $\bar{\theta} = 1$. Further, $\underline{\theta}$ increases with k and $\bar{\theta}$ decreases with k.*

It follows from Lemma 2 that the relationship between the expected posterior π^e and the cost-based thresholds $(e^c + k)$ and $(1 - e^c - k)$ can appear in three possible forms, as illustrated in Figure 1. In the first form, occurring if $k = 0$ and displayed in the left panel of Figure 1, $\pi^e \in [e^c, 1 - e^c]$ for all θ, and therefore only the litigation-trial regime prevails. In the second form, displayed in the middle panel of Figure 1, $0 < \underline{\theta} \leq \bar{\theta} < 1$, and there are three different regimes in equilibrium—the no-litigation regime for $\theta \in [0, \underline{\theta})$; the litigation-trial regime for $\theta \in [\underline{\theta}, \bar{\theta}]$; and the default-judgment regime for $\theta \in (\bar{\theta}, 1]$. Finally, in the third form, displayed in the right panel of Figure 1, $0 < \bar{\theta} < \underline{\theta} < 1$, and there are two different regimes in equilibrium—the no-litigation regime for $\theta \in [0, \bar{\theta})$; and the default-judgment regime for $\theta \in (\bar{\theta}, 1]$.
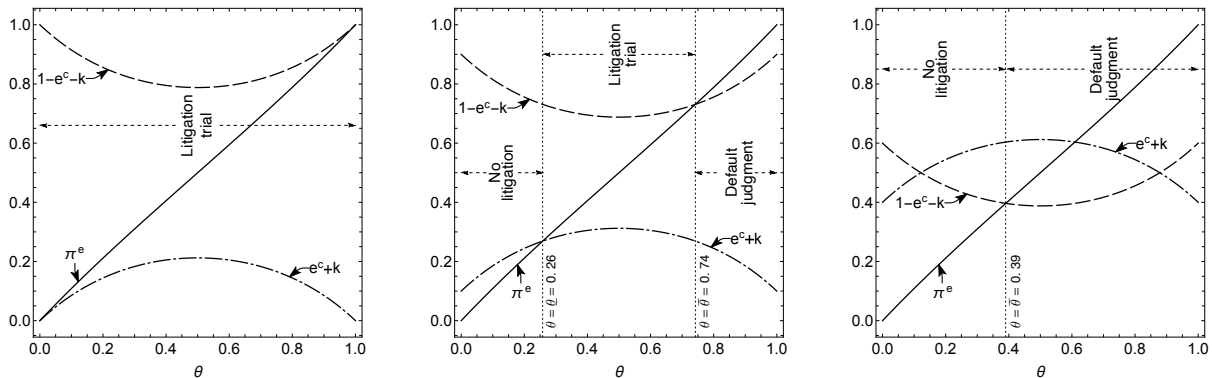

Figure 1: The expected posterior, the cost-based thresholds, and the equilibrium regimes against θ

Notes. All plots in Figure 1 consider the parameter values: $\mu = 0.9$, $h = 2$, $m = 0.5$, $\gamma = 0.75$. Further, we consider $k = 0$, 0.1, and 0.4, in the left-panel, middle-panel, and right-panel diagrams respectively. The continuous curve, the dot-dashed curve, and the dashed curve present π^e, $e^c + k$, and $1 - e^c - k$, respectively.

We can now easily compare the equilibrium regime between the two presumption scenarios—PV and NP. The presumption of validity moves the prior θ upward, i.e.,

from m to $\alpha + (1 - \alpha) m$, which takes a value in $(m, 1)$. As the no-litigation regime prevails for $\theta \in \left[0, \min \left\{ \underline{\theta}, \bar{\theta} \right\} \right)$ and the default-judgment regime prevails for $\theta \in \left(\bar{\theta}, 1 \right]$, it follows that an increase in θ will reduce the possibility of the no-litigation regime and increase the possibility of the default-judgment regime. The effect on the possibility of the litigation-trial regime is ambiguous: it depends on the relative extent of the bias in comparison to the thresholds $\underline{\theta}$ and $\bar{\theta}$.

The following proposition summarizes the above observations. The proof trivially follows from the preceding discussion.

Proposition 2. *Consider, as a point of comparison, NP as the default scenario. The introduction of the presumption of validity expands the set of parameter values for which the default-judgment regime prevails in equilibrium and shrinks the set of parameter values for which the no-litigation regime prevails in equilibrium. Further, the set of parameter values for which the litigation-trial regime prevails in equilibrium can either expand or shrink.*
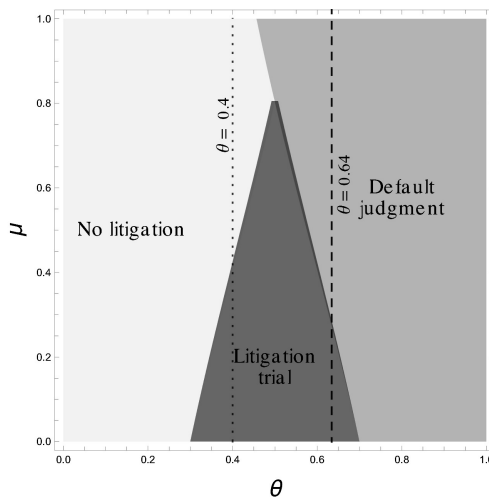

Figure 2: The equilibrium regimes in the (θ, μ) space

Notes. Figure 2 considers the parameter values: $h = 2$, $\gamma = 0.9$, $m = 0.4$ and $k = 0.3$. The equilibrium regimes are plotted in the (θ, μ) space. The dotted and the dashed line represent the priors $\theta = 0.4$ and $\theta = 0.64$, respectively.

Figure 2 plots the equilibrium regimes in the (θ, μ) space. To understand the findings of Proposition 2, consider a patent of merit $m = 0.4$ and $\alpha = 0.4$. If there is no presump-

tion, the prior is $\theta = m = 0.4$, which is represented by the dotted line in Figure 2. At $\theta = 0.4$, the litigation-trial regime exists for low values of μ. With the presumption of validity, the prior is $\theta = 0.4 + 0.6 \times 0.4 = 0.64$, which is represented by the dashed line in the figure. At $\theta = 0.64$, the default-judgment regime prevails over a larger range of values of μ and the no-litigation regime ceases to occur.

Proposition 2 shows that the presumption of validity affects the existence of the litigation-trial regime in equilibrium in an ambiguous way. To explore further, we next focus on two interesting features of the litigation-trial regime. First, a litigation trial is socially costly, mainly because resources are dissipated for non-productive activities. Second, there is a scope for learning through evidence produced in the trial. The learning moderates the possibility of making errors of judgment, i.e., rejecting the validity of a valid patent and accepting the validity of an invalid patent during a trial. In the following two subsections, we analyze the effects of the presumption of validity on these two features of the litigation-trial regime, namely resource dissipation and possible errors of judgment.

4.2   Resource dissipation

The total resource dissipation is given by

$$R = 2e^c + 2k = 2\theta\left(1 - \theta\right)\mu\Gamma + 2k.$$

The presumption criterion affects R only through the prior θ. We first study how a generic prior θ affects R for a fixed value of m. It follows from Lemma A.1 that R is increasing in $\theta \leq 1/\left(h^\mu + 1\right)$ and decreasing in $\theta \geq h^\mu/\left(h^\mu + 1\right)$. Further, under Assumption 1, R has its maximum at a unique θ, which we denote by θ^R. Specifically,

$$\theta^R := \arg\max_{\theta \in [0,1]} R \tag{16}$$

21

Under Assumption 1, it follows from the first-order condition of (16) that θ^R uniquely solves the following:

$$\left[\frac{q_{h1}h^\mu (h^\mu + 1)}{((1-\theta) + \theta h^\mu)^3} \left(\frac{1}{h^\mu + 1} - \theta \right) + 2q_0 \left(\frac{1}{2} - \theta \right) + \frac{q_{1h}h^\mu (h^\mu + 1)}{((1-\theta) h^\mu + \theta)^3} \left(\frac{h^\mu}{h^\mu + 1} - \theta \right) \right] = 0.$$

(17)

The following lemma documents the relationship between a patent's merit m and θ^R.

Lemma 3. *Fix m and consider R as a function of a generic prior θ. Under Assumption 1, R is uniquely maximized at some $\theta^R \in (0, 1)$. Further, $\theta^R < 1/2$ if $m > 1/2$, and $\theta^R > 1/2$ if $m < 1/2$. And, $\theta^R = m$ if $m = 1/2$.*

Figure 3 plots θ^R against m for a certain parametric specification. Suppose that the true state does not influence evidence production, i.e., $h = 1$. Then, it follows from (10) that $\Gamma = 1$ and $R = 2\theta (1 - \theta) \mu + 2k$, which is maximized at $\theta = 1/2$. Therefore, for every m, θ^R remains at $1/2$; in other words, if the true state did not influence the availability of favorable evidence, both parties expend their resources to the full extent when the court deems both states equally likely.
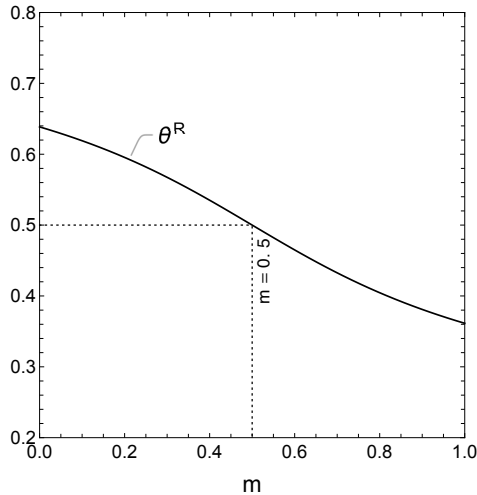

Figure 3: θ^R against m

Notes. Figure 3 considers the following parameter values: $\mu = 1$, $h = 2$, $\gamma = 0.9$.

To understand the findings of Lemma 3, consider what might happen if $h > 1$. We first argue for the case $m > 1/2$. As $h > 1$, P finds it easier to produce favorable

evidence compared to Q. The asymmetry discourages Q to expend resources, and, consequently, both parties' contest effort levels decrease. This discouragement effect can be partially mitigated if the court perceived the state of patent validity to be less likely, which would give Q an incentive to expend resources. This is the reason why θ^R, the resource-dissipation-maximizing prior value, must be less than $1/2$ whenever $m > 1/2$. Similarly, for $m < 1/2$, the asymmetric cost of evidence production discourages P to expend resources, and this discouragement effect can only be partially compensated if the court's prior is favorably biased toward the state of patent validity, resulting in $\theta^R > 1/2$.

Note that the presumption of validity not only changes the value of R, but also the type of the equilibrium regime. If, for some parametric specification, there is litigation-trial regime in equilibrium in one presumption scenario but not the other, then comparing resource dissipation between the two scenarios is straightforward. In contrast, if there is litigation trial in equilibrium under both PV and NP, then measuring the effect of the presumption on R is more complex.

To look into the effect of the presumption of validity, we now focus on the parameter space for which there will be litigation trial under both PV and NP and compare R between the two scenarios as m changes. Assume, without loss of generality, NP to be the default scenario and consider patents with $m > 1/2$. Then, $\theta^R < 1/2 < m$. The presumption of validity moves the value of the prior upward in the range $(m, 1)$. As R is decreasing in $\theta \in [\theta^R, 1]$, the presumption of validity will only reduce R. For $m < 1/2$, $m < 1/2 < \theta^R$. The effect on R is ambiguous; it can increase or decrease, depending on values of α and m, which determine to what extent the presumption-driven prior θ increases from m. Using the fact that R is concave under Assumption 1, the following proposition shows that the aggregate resource dissipation R is strictly lower under NP than under PV if and only if m is below a threshold lower than $1/2$.

Proposition 3. *Consider the range of parameter values for which the litigation-trial regime prevails in equilibrium in both the presumption scenarios, PV and NP. Further, consider, as a point of comparison, NP as the default scenario. There exists a threshold $m_{PV}^R \in (0, 1/2)$ such that the introduction of PV will increase (decrease) the aggregate*

resource dissipation R if m is less (greater) than m_{PV}^R.

The intuition behind Proposition 3 is straightforward. As m deviates from 1/2 in either direction, one of the two parties finds it easier to produce evidence, creating an imbalance that generally discourages both parties from investing. For high-merit patents, the presumption of validity exacerbates the existing imbalance. In contrast, for low-merit patents, the presumption partially mitigates the imbalance. Therefore, the threshold merit level at which the presumption can effectively neutralize the discouraging effects caused by asymmetric merit is always below 1/2.
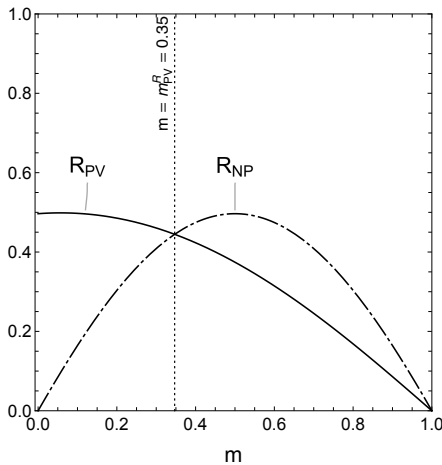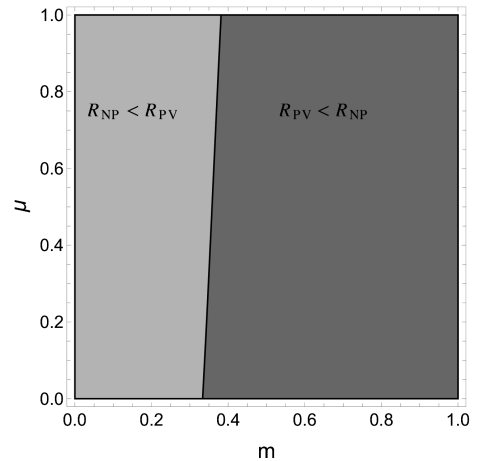

Figure 4: R against m

Figure 5: Comparison of R between PV and NP in the (m, μ) space

Notes. Figure 4 considers the parameter values: $k = 0$, $\mu = 1$, $h = 2$, $\gamma = 0.9$, and $\alpha = 0.5$. R_{PV} (the continuous curve) and R_{NP} (the dot-dashed curve) denote the resource dissipation R under $\theta = \theta_{PV}$ and $\theta = m$, respectively and we have $R_{NP} < R_{PV}$ if $m < m_{PV}^R = 0.35$. Figure 5 considers the same parameter values (relaxing μ) and compares R_{PV} with R_{NP} in the (m, μ) space.

In Figure 4, we plot R against m under PV and NP. Figure 5 compares R between PV and NP in the (m, μ) space. In Figures 4 and 5, we consider $k = 0$ so that the litigation-trial regime prevails in equilibrium for all possible priors under both PV and NP.

4.3 Error of judgment

An error of judgment in the court's decision-making occurs if the outcome of the litigation game results in either rejecting a patent's validity when it is valid (equivalent to a false negative or type-II error in statistical binary classification), or accepting a patent's validity when it is invalid (equivalent to a false positive or type-I error). The aggregate probability of making an error of judgment in litigation is given by[20]

$$E = \Pr[P \text{ wins } \cap s = I] + \Pr[Q \text{ wins } \cap s = V]$$

$$= \Pr[P \text{ wins } | s = I] \Pr[s = I] + \Pr[Q \text{ wins } | s = V] \Pr[s = V] \tag{18}$$

$$= m + \sum_{i \in \{h,1\}, j \in \{h,1\}} \pi(ie_p, je_Q, \theta) \begin{bmatrix} (1-m) \Pr[event_{ij} | s = I] \\ -m \Pr[event_{ij} | s = V] \end{bmatrix} \tag{19}$$

Using (2) and the fact that $e_P = e_Q = e^c$ in equilibrium, we can replace $\pi(ie_p, je_Q, \theta)$ by $\theta i^\mu / [(1-\theta)j^\mu + \theta i^\mu]$. Further, in $event_{hh}$ and $event_{11}$, we have $i = j$, and it follows that $\pi(ie_p, je_Q, \theta) = \theta$. Using the state-conditional probabilities of various events from (6), (19) further reduces to the following expression:

$$E = m + \frac{\theta h^\mu \left[(1-m)(1-\gamma)^2 - m\gamma^2\right]}{(1-\theta) + \theta h^\mu}$$
$$+ \frac{\theta \left[(1-m)\gamma^2 - m(1-\gamma)^2\right]}{(1-\theta)h^\mu + \theta} + 2\theta\left[(1-2m)\gamma(1-\gamma)\right]. \tag{20}$$

As shown in (18), E is a weighted sum of two conditional probabilities, measuring chances of a false positive ($\Pr[P \text{ wins } | s = I]$) and false negative ($\Pr[Q \text{ wins } | s = V]$). Further, the weights are not constant across merits. The weight associated with a false positive is high for low-merit patents and low for high-merit patents (see the Online Appendix for further discussion). The following lemma documents some useful properties of E.

Lemma 4. *Fix m and consider E as a function of a generic prior θ. Then,*

(i) There exists thresholds $\underline{m} < 1/2 < \overline{m}$ such that for any $m \leq \underline{m}$, E is increasing

---

[20]Here and in what follows, we slightly ease the notation by denoting the measure of judgment error as E without any subscript. Previously, we used E_i with a subscript i to indicate the evidence produced by party i.

in $\theta \in [0,1]$, and for any $m \geq \overline{m}$, E is decreasing in $\theta \in [0,1]$.

(ii) Further, for $m \leq 1/2$, E is increasing in θ for all $\theta \in [1/2,1]$ and for $m \geq 1/2$, E is decreasing in θ for all $\theta \in [0,1/2]$. It follows that for $m = 1/2$, E reaches its minimum at $\theta = 1/2$.

Part (i) of the above lemma illustrates the effect of the presumption criterion in clear terms when m takes sufficiently high or low values. For example, for $m \geq \overline{m}$, the presumption of validity only reduces an error of judgment by shifting the prior upward. The opposite effect will be realized for $m \leq \underline{m}$. For intermediate values of m, the analysis becomes complex.

A change in θ, after controlling for merit m, affects E in two distinct ways. The first is a *direct* effect. An increase in the prior θ leads to a qualitatively similar increase in the posterior π, which raises the probability of false positive and reduces the probability of a false negative. The second is an *indirect* effect. When θ gets close to $1/2$, because of the high intensity of competition, both parties spend a high volume of resources in unearthing new evidence. The probabilities of both a false positive and a false negative decrease with high evidence-seeking incentives.

To understand the direction of the combined effect, consider first the case of a low-merit patent, with m below $1/2$. Because a low-merit patent has higher weight on the false positive than on the false negative, the direct effect on E described above is increasing in θ. In contrast, the indirect effect due to evidence-seeking incentive does not follow a monotone path. As θ approaches $1/2$, the indirect effect dampens E, but then raises it as θ moves further away from $1/2$. Together, for sufficiently low values of m, the direct effect dominates the indirect because of the high weight on the conditional probability of false positive, and E increases with θ over its full range. For m close to $1/2$, the indirect effect can dominate the direct effect and E might be decreasing in $\theta \in (m, 1/2)$. However, for $\theta > 1/2$, both effects move in the same direction, which explains the second part of the above lemma. In the case of a high-merit patent with $m > 1/2$, the indirect effect works the same way as above but the direct effect works in the opposite direction. This is because a high-merit patent puts higher weight on the false negative than on the false

positive, and an increase in θ reduces the probability of a false negative and raises the probability of a false positive.

Next, to study the effect of the presumption of validity, we vary m and compare E between the two scenarios, PV and NP. Because a low-merit patent ($m < 1/2$) puts more weight on the probability of a false positive than a high-merit patent ($m > 1/2$) does, the direct effect of the presumption bias typically increases the error of judgment by a higher margin for a low-merit patent. However, when m is below but close to $1/2$, the evidence-seeking incentive is high even without the presumption bias, and the marginal effect of increasing the competition intensity by biasing the prior is relatively low. Thus, the positive indirect effect of the bias by increasing the evidence-seeking incentive is always dominated by the negative direct effect of increasing the probability of a false positive for all patents with $m < 1/2$. The following proposition shows that for all m below a threshold that is weakly higher than $1/2$, PV is always associated with a higher error of judgment compared to NP.

Proposition 4. *Consider the range of parameter values for which the litigation-trial regime prevails in equilibrium in both the presumption scenarios, PV and NP. Further, consider, as a point of comparison, NP as the default scenario. There exists a threshold $m_{PV}^E \in (1/2, 1)$ such that the introduction of PV will increase the error of judgment E if m is less than m_{PV}^E.*

It is worth noting that unlike Proposition 3, Proposition 4 does not provide a necessary and sufficient threshold-based result for comparing E between the two regimes. This is because the two effects of the presumption bias described earlier, the direct effect of changing probabilities of a false positive and false negative and the indirect effect of changing evidence-seeking incentive, can move at different rates causing E to change in a non-monotone way.

Figures 6 and 7 illustrate the proposition's findings using a numerical example. In Figure 6, we plot E against m under PV and NP. Figure 7 compares E between PV and NP in the (m, μ) space. We consider $k = 0$ so that the litigation-trial regime prevails in equilibrium for all possible priors under both PV and NP.
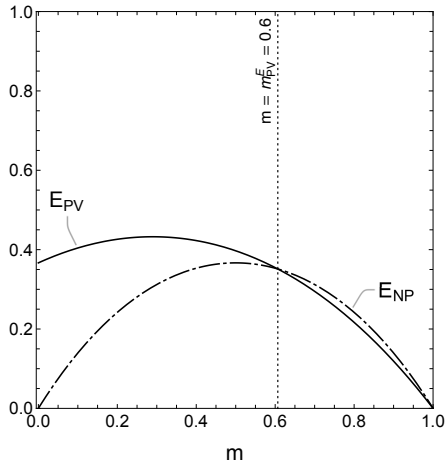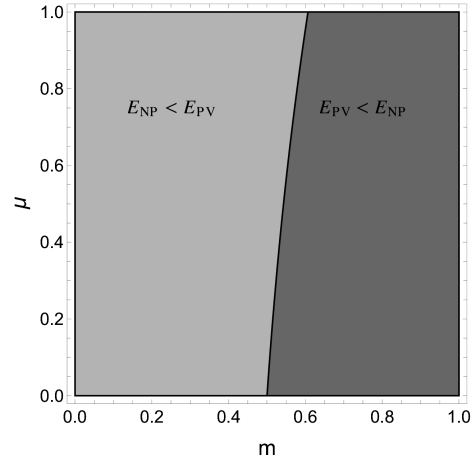
Figure 6: E against m

Figure 7: Comparison of E between PV
and NP

Notes. Figure 6 considers the parameter values: $k = 0$, $\mu = 1$, $h = 2$, $\gamma = 0.9$, and $\alpha = 0.5$. E_{PV} (the continuous curve) and E_{NP} (the dot-dashed curve) denote the error value E, computed at the priors $\theta = \theta_{PV}$ and $\theta = m$, respectively and we have $E_{NP} < E_{PV}$ if $m < m_{PV}^E = 0.6$. Figure 7 considers the same parameter values (relaxing μ), and compares E_{PV} with E_{NP} in the (m, μ) space.

5 Conclusion

A litigation trial is a non-productive contest. It only results in a transfer of ownership rights. The notion of economic efficiency is built around the concept of allocative efficiency, which deals with how costly resources can be optimally allocated to productive activities in order to achieve maximum benefits. By this notion, spending resources in any non-productive contest is an inefficient activity (Tullock, 1967; Krueger, 1974; Bhagwati, 1982). Our framework, however, explores a different effect of investing resources in a litigation contest. It helps unearthing new evidence that contributes to better judgment and proper allocation of ownership rights. An error of judgment is costly to society, just as resource dissipation is (Buzzacchi and Scellato, 2008). An evaluation of the presumption criterion should therefore include both dimensions, resource dissipation and error of judgment.

In this context, the findings of Propositions 3 and 4 reflect the potential trade-off associated with the presumption criterion when there is sufficient uncertainty about the

patent's merit. In particular, for patents with merit m between the two thresholds, m_{PV}^R and m_{PV}^E, placing on either side of $1/2$, the presumption has contrasting effects. In this case, introducing the presumption will likely raise the error of judgment, but decrease the resource dissipation. This is because, for m close to $1/2$, the intensity of competition is at its peak. The bias induced by the presumption likely reduces the intensity, because of which fewer resources would be dissipated. However, it would also reduce the incentive to gather new evidence, which would rather be highly needed especially when there is great uncertainty about the patent's merit.

This finding suggests that we should pay careful attention to the application of the presumption, especially in contexts where the examination of patent applications is complex and invalid patents are granted more frequently by the patent office (e.g., high technology sectors). Examining patent applications is becoming increasingly difficult for several reasons, including the growing number of applications and the budgetary constraints the patent office faces. Granting of invalid patents is an ever-growing reality and the possibility of resolving disputes through legal proceedings is an inevitable consequence (Buzzacchi and Scellato, 2008; Farrell and Shapiro, 2008; de Rassenfosse et al., 2021). Accordingly, in these contexts where m is close to $1/2$, our findings support the arguments against the application of the presumption, which should not be accepted, or even dismissed *tout court*.

Our analysis provides novel insights into how the presumption of validity affects the frequency of litigation trials, and reveals its countervailing efficiency effects when there is uncertainty about the patent's merit. Our findings come from analyzing partial effects, under the maintained hypothesis that the patent has been produced and granted. Some significant factors are absent from our analysis, which can be extended in various directions. Future research can analyze whether changes in presumption criteria affect the level of R&D investments and the frequency of patent applications, and what implications can be derived for growth and innovation. Also, it could be interesting to study the effects of litigation outcomes on the market structure, under different presumption criteria. In some situations, a favorable outcome to the plaintiff or the defendant can

increase or decrease the market competition, with additional gain or loss in consumer surplus. Finally, our setup can be extended to analyze the interaction of legal presumptions with other institutional variables, including the standard of proof and the structure of the patent-granting process.

Appendix A

This Appendix contains the proofs that are omitted in the main text. We will begin with documentation of three additional results, Lemmas A.1, A.2, and A.3, that will be useful in proving our main findings. Proofs of these additional lemmas are provided in the online Appendix B.

Lemma A.1. *The equilibrium expenditure, e^c, is increasing in $\theta \in [0, 1/(h^\mu + 1)]$ and decreasing in $\theta \in [h^\mu/(h^\mu + 1), 1]$. Further, if $h^\mu \le 2$, then e^c is concave in $\theta \in [0, 1]$.*

Lemma A.2. *Define $F(\theta) := \pi^e - (1 - e^c - k)$. $F(\theta)$ is strictly increasing in $\theta \in [0, 1]$.*

Lemma A.3. *Define $G(\theta) := \pi^e - e^c - k$. $G(\theta)$ is strictly increasing in $\theta \in [0, 1]$.*

Proof of Lemma 1. At the trial stage, the participation cost is sunk, and therefore it does not affect the optimal choice of resource spending. The first-order condition of maximizing U_P with respect to e_P gives

$$\frac{q_{h1}\theta(1-\theta)\mu h (he_P)^{\mu-1}(e_Q)^\mu}{((1-\theta)(e_Q)^\mu + \theta(he_P)^\mu)^2} + \frac{q_{1h}\theta(1-\theta)\mu(e_P)^{\mu-1}(he_Q)^\mu}{((1-\theta)(he_Q)^\mu + \theta(e_P)^\mu)^2}$$
$$+ \frac{q_0\theta(1-\theta)\mu(e_P)^{\mu-1}(e_Q)^\mu}{((1-\theta)(e_Q)^\mu + \theta(e_P)^\mu)^2} = 1. \qquad (A.1)$$

Similarly, the first-order condition of maximizing U_Q with respect to e_Q gives

$$\frac{q_{h1}\theta(1-\theta)\mu(he_P)^\mu(e_Q)^{\mu-1}}{((1-\theta)(e_Q)^\mu + \theta(he_P)^\mu)^2} + \frac{q_{1h}\theta(1-\theta)\mu h(e_P)^\mu(he_Q)^{\mu-1}}{((1-\theta)(he_Q)^\mu + \theta(e_P)^\mu)^2}$$
$$+ \frac{q_0\theta(1-\theta)\mu(e_P)^\mu(e_Q)^{\mu-1}}{((1-\theta)(e_Q)^\mu + \theta(e_P)^\mu)^2} = 1. \qquad (A.2)$$

There is a symmetric solution $e_P = e_Q = e^c$, satisfying the two first-order conditions simultaneously, such that

$$\frac{q_{h1}\theta(1-\theta)\mu h^\mu}{((1-\theta) + \theta h^\mu)^2} + \frac{q_{1h}\theta(1-\theta)\mu h^\mu}{((1-\theta)h^\mu + \theta)^2} + q_0\theta(1-\theta)\mu = e^c. \qquad (A.3)$$

After simplifying, (A.3) reduces to $e^c = \theta(1-\theta)\mu\Gamma(m, \gamma, h, \mu, \theta)$, where $\Gamma(m, \gamma, h, \mu, \theta)$ is given by (10). It can be shown that the solution of the first-order condition also satisfies

the second-order condition when $\mu \leq 1$. To see this, note that P's expected payoff, U_P, is a linear combination of terms of the form $xe_P^\mu / \left(ye_Q^\mu + xe_P^\mu \right)$ for different expressions of $x, y > 0$, with positive coefficients, and $(-e_P)$. The second-order derivative of U_P will, therefore, be a linear combination of terms of the following forms with positive coefficients:

$$\frac{d^2}{de_P^2} \frac{xe_P^\mu}{\left(ye_Q^\mu + xe_P^\mu \right)} = \frac{xy\mu e_P^{2\mu-2} e_Q^\mu \left[y \left(\mu - 1 \right) - x \left(\mu + 1 \right) \right]}{\left(ye_Q^\mu + xe_P \right)^3},$$

which is strictly negative for any $x, y > 0$ if $e_p > 0$, $e_Q > 0$, and $\mu \leq 1$. Therefore, U_P is globally concave for $e_p > 0$, $e_Q > 0$.

Similarly, Q's expected payoff, U_Q, is a linear combination of terms of the form $xe_P^\mu / \left(ye_Q^\mu + xe_P^\mu \right)$ for different expressions of $x, y > 0$, with negative coefficients, and $(-e_Q)$. Therefore, the second-order derivative of U_Q will be a linear combination of terms of the following forms with negative coefficients:

$$\frac{d^2}{de_Q^2} \frac{xe_P^\mu}{\left(ye_Q^\mu + xe_P^\mu \right)} = \frac{xy\mu e_P^\mu e_Q^{2\mu-2} \left[y \left(\mu + 1 \right) + x \left(1 - \mu \right) \right]}{\left(ye_Q^\mu + xe_P \right)^3},$$

which is strictly positive for any $x, y > 0$ if $e_p > 0$, $e_Q > 0$, and $\mu \leq 1$. Because of the negative coefficients, the second-order derivative of U_Q is, therefore, negative and so U_Q is globally concave for $e_p > 0$, $e_Q > 0$. Further, the first-order conditions (A.1) and (A.2) being strictly positive at $(e_P = 0, e_Q = e^c)$ and $(e_P = e^c, e_Q = 0)$, respectively, implying that the solution e^c is indeed a global maxima for each player, given the other player plays e^c. $\qquad \square$

Proof of Lemma 2. It follows from Lemma A.3 that $G\left(\theta \right) = \pi^e - e^c - k$ is strictly increasing in θ; and the continuity of $G\left(\theta \right)$ implies that we can find a threshold $\underline{\theta}$, which is increasing in k, such that $G\left(\theta \right) \geq 0$ if and only if $\theta \geq \underline{\theta}$. Further, $G\left(0 \right) = -k$, and therefore, $\underline{\theta} = 0$ if $k = 0$. Similarly, Lemma A.2 finds that $F\left(\theta \right) = \pi^e - \left(1 - e^c - k \right)$ is increasing and its continuity with respect to θ implies the existence of a threshold $\overline{\theta}$ such that $F\left(\theta \right) \leq 0$ if and only if $\theta \leq \overline{\theta}$. Further, $F\left(1 \right) = k$, and therefore, $\overline{\theta} = 1$ if $k = 0$. $\quad \square$

Proof of Lemma 3. Because $R = 2k$ at $\theta = 0, 1$, and because of the concavity of e^c by Assumption 1, $R > 2k$ for all $\theta \in (0, 1)$ and $\theta^R \in (0, 1)$ uniquely solves $dR/d\theta = 0$, which is equivalent to $de^c/d\theta = 0$ and can be expressed as (17).

Further, because of the concavity of e^c, $\theta^R < 1/2$ if the left-hand-side of (17) is strictly negative at $\theta = 1/2$, and $\theta^R > 1/2$ if the left-hand-side of 17 is strictly positive at $\theta = 1/2$. The left-hand-side of (17), when computed at $\theta = 1/2$, is given by

$$\frac{8q_{h1}h^\mu (h^\mu + 1)}{(h^\mu + 1)^3} \left(\frac{1}{h^\mu + 1} - \frac{1}{2} \right) + \frac{8q_{1h}h^\mu (h^\mu + 1)}{(h^\mu + 1)^3} \left(\frac{h^\mu}{h^\mu + 1} - \frac{1}{2} \right)$$
$$= \frac{4q_{h1}h^\mu (1 - h^\mu)}{(h^\mu + 1)^3} + \frac{8q_{1h}h^\mu (h^\mu - 1)}{(h^\mu + 1)^3}$$
$$= -\frac{4h^\mu (h^\mu - 1)}{(h^\mu + 1)^3} [q_{h1} - q_{1h}]$$
$$= -\frac{16h^\mu (h^\mu - 1)}{(h^\mu + 1)^3} \left(\gamma - \frac{1}{2} \right) \left(m - \frac{1}{2} \right),$$

which is strictly negative if $m > 1/2$, and is strictly positive if $m < 1/2$. Therefore, $\theta^R < 1/2$ if $m > 1/2$, and $\theta^R > 1/2$ if $m < 1/2$. Finally, if $m = 1/2$, then $\theta = 1/2$ is a solution of (17), implying $\theta^R = 1/2$. $\qquad \square$

Proof of Proposition 3. We let θ_{PV} and θ_{NP} denote the priors under the presumption of validity and the presumption of no validity, respectively. Therefore, $\theta_{PV} = \alpha + (1 - \alpha) m$ and $\theta_{NP} = m$. We let R_{PV} and R_{NP} denote the value of R under the presumption of validity and the presumption of no validity, respectively. Specifically, $R_{PV} = R(\theta_{PV})$ and $R_{NP} = R(\theta_{NP})$. Note that $R(0) = R(1) = 2k$, and by Assumption 1, R is concave in θ with a maximum at θ^R.

First, consider $m \geq 1/2$. Then, by Lemma 3, $\theta^R \leq 1/2$ and by concavity, θ^R is decreasing in $\theta \in [\theta^R, 1]$. Further, as $\theta_{PV} > \theta_{NP} = m \geq \theta^R$, we have $R(\theta_{PV}) < R(\theta_{NP})$. Hence, for all $m \geq 1/2$, $R_{PV} < R_{NP}$.

Next, we consider $m < 1/2$. Then, $\theta^R > 1/2$ and therefore, $m < \theta^R$. As $R(\theta)$ is concave with a unique maximum at θ^R, it follows that for every $\theta < \theta^R$, there exists some $f(\theta) \in [\theta^R, 1]$ such that $R(\theta) = R(f(\theta))$, $R(\theta) < R(\theta')$ for all $\theta' \in (\theta, f(\theta))$ and

$R(\theta) > R(\theta')$ for all $\theta' \in (f(\theta), 1]$. Further, observe that the mapping $f(\theta)$ is decreasing in $\theta < \theta^R$, which follows from the fact that $R(\theta)$ is decreasing in the range $[\theta^R, 1]$.

Now, consider some $m < 1/2$ such that $R_{NP}(m) = R(\theta_{NP}(m)) < R(\theta_{PV}(m)) = R_{PV}(m)$, which is equivalent to $m < \theta_{PV}(m) < f(m)$ by construction of f. Then, for all $m' < m$, we have $\theta_{PV}(m') < \theta_{PV}(m)$ because θ_{PV} is increasing in m, and $f(m') > f(m)$ because f is decreasing in $m < \theta^R$. Together, it follows that $\theta_{PV}(m') < \theta_{PV}(m) < f(m) < f(m')$ and consequently, $R_{NP}(m') = R(\theta_{NP}(m')) < R(\theta_{PV}(m')) = R_{PV}(m')$. Next, consider some $m < 1/2$ such that $R_{NP}(m) > R_{PV}(m)$, which is equivalent to $f(m) < \theta_{PV}(m)$ by construction of f. Then, for all $m' \in (m, 1/2)$, we have $\theta_{PV}(m) < \theta_{PV}(m')$ because θ_{PV} is increasing in m, and $f(m') < f(m)$ because f is decreasing in $m < \theta^R$. Together, it follows that $f(m') < f(m) < \theta_{PV}(m) < \theta_{PV}(m')$ and consequently, $R_{PV}(m') = R(\theta_{PV}(m')) < R(\theta_{NP}(m')) = R_{NP}(m')$.

We have just shown that for any $m < 1/2$, if $R_{NP}(m) < R_{PV}(m)$, then for all $m' < m$, $R_{NP}(m') < R_{PV}(m')$, and if $R_{PV}(m) < R_{NP}(m)$, then for all $m' > m$, $R_{PV}(m') < R_{NP}(m')$. This observation, together with the fact $R_{PV}\mid_{m=1/2} < R_{NP}\mid_{m=1/2}$, implies that $R_{NP} < R_{PV}$ if and only if m is less than some threshold and the threshold is less than $1/2$. We refer to this threshold by m_{PV}^R. $\qquad\square$

Proof of Lemma 4. To prove part (i), consider the derivative of E with respect to θ:

$$\frac{dE(\theta)}{d\theta} = \frac{q_1 h^\mu}{((1-\theta) + \theta h^\mu)^2} + q_2 + \frac{q_3 h^\mu}{((1-\theta) h^\mu + \theta)^2},$$

where

$$q_1 = (1-m)(1-\gamma)^2 - m\gamma^2 = (1-\gamma)^2 - m\left[(1-\gamma)^2 + \gamma^2\right],$$

$$q_2 = 2(1-2m)\gamma(1-\gamma),$$

$$q_3 = (1-m)\gamma^2 - m(1-\gamma)^2 = \gamma^2 - m\left[(1-\gamma)^2 + \gamma^2\right].$$

Observe that q_1, q_2, and q_3 are linear and decreasing in m. Further, as $1-\gamma < \gamma$, when $m < 1/2$, $q_2 > 0$, $q_3 > 0$, and for sufficiently low values of m, $q_1 > 0$. It follows that there

is a threshold value $\underline{m} < 1/2$ such that for any $m \leq \underline{m}$, all q_1, q_2, and q_3 are positive and $dE(\theta)/d\theta > 0$ for all $\theta \in [0,1]$. Similarly, when $m > 1/2$, $q_2 < 0$, $q_1 < 0$, and for sufficiently high values of m, $q_3 < 0$. Therefore, there is a threshold value $\overline{m} > 1/2$ such that for any $m \geq \overline{m}$, all q_1, q_2, and q_3 are negative and $dE(\theta)/d\theta < 0$ for all $\theta \in [0,1]$. This completes the proof of part (i) of the lemma.

To prove part (ii), observe that $q_1 < q_3$ and

$$h^\mu / \left((1-\theta) + \theta h^\mu\right)^2 \lessgtr h^\mu / \left((1-\theta) h^\mu + \theta\right)^2 \Leftrightarrow \frac{1}{2} \lessgtr \theta.$$

Let us first consider $m \leq 1/2$ and $\theta \geq 1/2$. From $m \leq 1/2$, it follows that $q_2 \geq 0$ and $q_3 > 0$. If $q_1 \geq 0$, then it trivially follows that $dE(\theta)/d\theta \geq 0$. Consider the possibility that $q_1 < 0$. Further, $m \leq 1/2$ implies that $-q_3 < q_1$, or, equivalently, $q_1 + q_3 > 0$. From $\theta \geq 1/2$, it follows that $h^\mu / \left((1-\theta) + \theta h^\mu\right)^2 \leq h^\mu / \left((1-\theta) h^\mu + \theta\right)^2$. Applying these observations, we get

$$dE(\theta)/d\theta = \frac{q_1 h^\mu}{\left((1-\theta) + \theta h^\mu\right)^2} + q_2 + \frac{q_3 h^\mu}{\left((1-\theta) h^\mu + \theta\right)^2} \geq \frac{h^\mu (q_1 + q_3)}{\left((1-\theta) + \theta h^\mu\right)^2} + q_2 \geq 0.$$

Next, consider $m \geq 1/2$ and $\theta \leq 1/2$. From $m \geq 1/2$, it follows that $q_2 \leq 0$, $q_1 < 0$. If $q_3 \leq 0$, it trivially follows that $dE^{LT}(\theta)/d\theta \leq 0$. Consider the possibility that $q_3 > 0$. Further, $m \geq 1/2$ implies that $q_1 < -q_3$, or, equivalently, $q_1 + q_3 < 0$. From $\theta \leq 1/2$, it follows that $h^\mu / \left((1-\theta) + \theta h^\mu\right)^2 \geq h^\mu / \left((1-\theta) h^\mu + \theta\right)^2$. Applying these observations, we get q_2

$$dE(\theta)/d\theta = \frac{q_1 h^\mu}{\left((1-\theta) + \theta h^\mu\right)^2} + q_2 + \frac{q_3 h^\mu}{\left((1-\theta) h^\mu + \theta\right)^2} \leq \frac{h^\mu (q_1 + q_3)}{\left((1-\theta) + \theta h^\mu\right)^2} + q_2 \leq 0.$$

When $m = 1/2$, the analysis above suggests that $E(\theta)$ is increasing in θ for $\theta \geq 1/2$ and is decreasing in θ for $\theta \leq 1/2$, implying that $E(\theta)$ reaches its minimum at $\theta = 1/2$. This completes the proof of part (ii) of the lemma. $\qquad \square$

Proof of Proposition 4. We let θ_{PV} and θ_{NP} denote the priors under the presumption of validity and the presumption of no validity, respectively. Therefore, $\theta_{PV} =$

$\alpha + (1 - \alpha) m$ and $\theta_{NP} = m$. Similarly, we let E_{PV} and E_{NP} denote the value of E under the presumption of validity and the presumption of no validity, respectively. Specifically, $E_{PV} = E(\theta_{PV})$ and $E_{NP} = E(\theta_{NP})$. To prove the part (i) of the proposition, we will show that $\Delta_{PV} := E_{PV} - E_{NP} \geq 0$ for all $m \leq 1/2$.

Consider the range of parameter values such that the litigation trial regime prevails in equilibrium in both the presumption scenarios, PV and NP. Therefore, using (20), $\Delta_{PV} = E(\theta_{PV}) - E(m)$ can be expressed as

$$\Delta_{PV} = h^\mu \alpha (1 - m) \left[\frac{q_1}{ab} + \frac{q_3}{cd} + \frac{q_2}{h^\mu} \right],$$

where

$$q_1 = (1 - \gamma)^2 - m \left(\gamma^2 + (1 - \gamma)^2 \right), \, q_2 = 2\gamma (1 - \gamma)(1 - 2m),$$

$$q_3 = \gamma^2 - m \left(\gamma^2 + (1 - \gamma)^2 \right), \, a = (1 - \theta_{PV}) + \theta_{PV} h^\mu,$$

$$b = (1 - m) + m h^\mu, \, c = (1 - m) h^\mu + m, \, d = (1 - \theta_{PV}) h^\mu + \theta_{PV}.$$

Observe that for $m \leq 1/2$, $q_2 \geq 0$ and $q_3 > 0$.

For $m \leq (1 - \gamma)^2 / \left(\gamma^2 + (1 - \gamma)^2 \right) < 1/2$, we have $q_1 \geq 0$, and it therefore trivially follows that $\Delta_{PV} \geq 0$.

For $m \in \left((1 - \gamma)^2 / \left(\gamma^2 + (1 - \gamma)^2 \right), 1/2 \right]$, we have $q_1 < 0$, $q_2 \geq 0$, and $q_3 > 0$. In this range of values of m, $(q_1/ab) + (q_3/cd) \geq 0$ implies $\Delta_{PV} \geq 0$.

Claim 1: $(q_1/ab) + (q_3/cd) \geq 0$ at $m = 1/2$.

Proof of Claim 1: At $m = 1/2$, we have $a > b = (h^\mu + 1)/2 = c > d$ and $q_3 = -q_1 = \gamma^2 - (1 - \gamma)^2$. Therefore,

$$(q_1/ab) + (q_3/cd) = \frac{2 \left(\gamma^2 - (1 - \gamma)^2 \right)}{h^\mu + 1} \left(\frac{1}{d} - \frac{1}{a} \right) > 0,$$

which proves Claim 1.

Claim 2: (q_1/ab) is decreasing in m for $m \in \left((1 - \gamma)^2 / \left(\gamma^2 + (1 - \gamma)^2 \right), 1/2 \right]$.

Proof of Claim 2: Consider the derivative of (q_1/ab) (for brevity, we write df/dm as

f' when f is a function of m):

$$\frac{d}{dm}\frac{q_1}{ab} = \frac{(q_1)'\,ab - q_1\,(ab)'}{(ab)^2} \le 0 \Leftrightarrow (q_1)'\,ab \le q_1\,(ab)' \Leftrightarrow \frac{(q_1)'}{q_1} \ge \frac{(ab)'}{ab}. \tag{A.4}$$

The inequality is reversed in the last part of the above chain of equivalent conditions because $q_1 < 0$. Note that

$$\frac{(q_1)'}{q_1} = \frac{\left(\gamma^2 + (1-\gamma)^2\right)}{m\left(\gamma^2 + (1-\gamma)^2\right) - (1-\gamma)^2} > 0$$

for $m \in \left((1-\gamma)^2 / \left(\gamma^2 + (1-\gamma)^2\right), 1/2\right]$ and is decreasing in m. Therefore, its minimum value is reached at $m = 1/2$ and

$$\min_{m \in \left(\frac{(1-\gamma)^2}{\gamma^2 + (1-\gamma)^2}, 1/2\right]} \frac{(q_1)'}{q_1} = \frac{2\left(\gamma^2 + (1-\gamma)^2\right)}{\gamma^2 - (1-\gamma)^2} > 2. \tag{A.5}$$

Further,
$$\frac{(ab)'}{ab} = \frac{a'}{a} + \frac{b'}{b} = \frac{(1-\alpha)\left(h^\mu - 1\right)}{(1-\theta_{PV}) + \theta_{PV}h^\mu} + \frac{\left(h^\mu - 1\right)}{(1-m) + mh^\mu},$$

and each of the two terms is less than 1 by Assumption 1. Therefore,

$$\max_{m \in \left(\frac{(1-\gamma)^2}{\gamma^2 + (1-\gamma)^2}, 1/2\right]} \frac{(ab)'}{ab} \le 2. \tag{A.6}$$

Therefore, (A.4) must hold for $m \in \left((1-\gamma)^2 / \left(\gamma^2 + (1-\gamma)^2\right), 1/2\right]$, and therefore, (q_1/ab) is decreasing in this range of values of m, which completes the proof of Claim 2.

Claim 3: (q_3/cd) is decreasing in m for $m \in \left((1-\gamma)^2 / \left(\gamma^2 + (1-\gamma)^2\right), 1/2\right]$.

Proof of Claim 3: Consider the derivative of (q_3/cd):

$$\frac{d}{dm}\frac{q_3}{cd} = \frac{(q_3)'\,cd - q_3\,(cd)'}{(cd)^2} \le 0$$

$$\Leftrightarrow (q_3)'\,cd \le q_3\,(cd)' \Leftrightarrow \frac{(q_3)'}{q_3} \le \frac{(cd)'}{cd} = \frac{c'}{c} + \frac{d'}{d}. \tag{A.7}$$

Observe that

$$\frac{(q_3)'}{q_3} = -\frac{\left(\gamma^2 + (1-\gamma)^2\right)}{\gamma^2 - m\left(\gamma^2 + (1-\gamma)^2\right)},$$

$$\frac{c'}{c} = -\frac{(h^\mu - 1)}{h^\mu - m(h^\mu - 1)},$$

$$\frac{d'}{d} = -\frac{(1-\alpha)(h^\mu - 1)}{h^\mu - \theta_{PV}(h^\mu - 1)},$$

and, therefore, (A.7) can be rewritten as

$$\frac{(h^\mu - 1)}{h^\mu - m(h^\mu - 1)} + \frac{(1-\alpha)(h^\mu - 1)}{h^\mu - \theta_{PV}(h^\mu - 1)} \leq \frac{\left(\gamma^2 + (1-\gamma)^2\right)}{\gamma^2 - m\left(\gamma^2 + (1-\gamma)^2\right)}. \qquad \text{(A.8)}$$

Further, note that

$$\frac{(1-\alpha)(h^\mu - 1)}{h^\mu - \theta_{PV}(h^\mu - 1)} \leq \frac{(h^\mu - 1)}{h^\mu - m(h^\mu - 1)}$$

$$\Leftrightarrow (1-\alpha)h^\mu - (1-\alpha)m(h^\mu - 1) \leq h^\mu - \theta_{PV}(h^\mu - 1)$$

$$\Leftrightarrow \alpha(h^\mu - 1) \leq \alpha h^\mu,$$

which holds true. Therefore, to prove (A.8), it is sufficient to show that

$$\frac{2(h^\mu - 1)}{h^\mu - m(h^\mu - 1)} \leq \frac{\left(\gamma^2 + (1-\gamma)^2\right)}{\gamma^2 - m\left(\gamma^2 + (1-\gamma)^2\right)}.$$

We can simplify the above inequality as follows:

$$\frac{2(h^\mu - 1)}{h^\mu - m(h^\mu - 1)} \leq \frac{\left(\gamma^2 + (1-\gamma)^2\right)}{\gamma^2 - m\left(\gamma^2 + (1-\gamma)^2\right)}$$

$$\Leftrightarrow 2(h^\mu - 1)(1-m)\gamma^2 - 2(h^\mu - 1)m(1-\gamma)^2$$

$$\leq \gamma^2(h^\mu - m(h^\mu - 1)) + (1-\gamma)^2(h^\mu - m(h^\mu - 1))$$

$$\Leftrightarrow -(1-\gamma)^2(2(h^\mu - 1)m + h^\mu - m(h^\mu - 1))$$

$$\leq \gamma^2(h^\mu - m(h^\mu - 1) - 2(h^\mu - 1)(1-m))$$

$$\Leftrightarrow -(1-\gamma)^2((h^\mu - 1)m + h^\mu) \leq \gamma^2(2 - h^\mu + m(h^\mu - 1)),$$

38

which holds true, because the right-hand side is positive by Assumption 1 and the left-hand side is negative. This proves Claim 3.

Claims 1, 2, and 3 show that $(q_1/ab) + (q_3/cd) \geq 0$ at $m = 1/2$ and $(q_1/ab) + (q_3/cd)$ is decreasing in m for $m \in \left((1-\gamma)^2 / \left(\gamma^2 + (1-\gamma)^2 \right), 1/2 \right]$. Therefore, $(q_1/ab) + (q_3/cd) \geq 0$, and consequently, $\Delta_{PV} \geq 0$ for $m \in \left((1-\gamma)^2 / \left(\gamma^2 + (1-\gamma)^2 \right), 1/2 \right]$. As we have already shown that $\Delta_{PV} \geq 0$ for $m \leq (1-\gamma)^2 / \left(\gamma^2 + (1-\gamma)^2 \right)$, it follows that $\Delta_{PV} \geq 0$ for $m \leq 1/2$.

At $m = 1$, $\theta_{PV} = m$, and $\Delta_{PV} = E^{LT}(\theta_{PV}) - E^{LT}(m) = 0$. Δ_{PV} can be negative for $m > 1/2$; however, because of continuity, we can find a threshold $m_{PV}^E \geq 1/2$ such that $\Delta_{PV} \geq 0$ if $m \leq m_{PV}^E$. Further, note that the result provides only a sufficient condition, but not a necessary condition, because Δ_{PV} is not monotone in m. $\qquad\square$

References

American Intellectual Property Law Association (2013). *Report of the Economic Survey.* Washington, DC: American Intellectual Property Law Association.

Austen-Smith, D. and J. R. Wright (1992). Competitive lobbying for a legislator's vote. *Social Choice and Welfare 9*(3), 229–257.

Bennedsen, M. and S. E. Feldmann (2002). Lobbying legislatures. *Journal of Political Economy 110*(4), 919–946.

Bennedsen, M. and S. E. Feldmann (2006). Informational lobbying and political contributions. *Journal of Public Economics 90*(4-5), 631–656.

Bernardo, A. E., E. Talley, and I. Welch (2000). A theory of legal presumptions. *The Journal of Law, Economics, & Organization 16*(1), 1–49.

Bhagwati, J. N. (1982). Directly Unproductive, Profit-Seeking (DUP) Activities. *Journal of Political Economy 90*(5), 988–1002.

Bock, J. W. (2014). Does the Presumption of Validity Matter-An Experimental Assessment. *University of Richmond Law Review 49*(2), 417–464.

Bohrer, D. C. (2004). Knocking the eagle off the patent owner's shoulder: Chiron holds that jurors don't have to be told that a patent is presumed valid. *Santa Clara Computer and High Technology Law Journal 21*(1), 259–288.

Buzzacchi, L. and G. Scellato (2008). Patent litigation insurance and R&D incentives. *International Review of Law and Economics 28*(4), 272–286.

Chan, J. and M. Fawcett (2005). Footsteps of the patent troll. *Intellectual Property Law Bulletin 10*(1), 1–12.

Chatlynne, E. S. (2010). Investigating Patent Law's Presumption of Validity–An Empirical Analysis. *Patently-O Patent Law Journal 2010*, 37–43.

Choi, J. P. and H. Gerlach (2015). Patent pools, litigation, and innovation. *The RAND Journal of Economics 46*(3), 499–523.

Chowdhury, S. M., P. Esteve-González, and A. Mukherjee (2023). Heterogeneity, leveling the playing field, and affirmative action in contests. *Southern Economic Journal 89*(3), 924–974.

Clark, D. J. and T. Kundu (2021). Competitive balance: Information disclosure and discrimination in an asymmetric contest. *Journal of Economic Behavior & Organization 184*, 178–198.

Clermont, K. M. and E. Sherwin (2002). A comparative view of standards of proof. *American Journal of Comparative Law 50*(2), 243–276.

Commission, F. T. (2004). To promote innovation: The proper balance of competition and patent law and policy: Executive summary. *Berkeley Technology Law Journal 19*(3), 861–883.

Cremers, K., M. Ernicke, F. Gaessler, D. Harhoff, C. Helmers, L. McDonagh, P. Schliessler, and N. Van Zeebroeck (2017). Patent litigation in Europe. *European Journal of Law and Economics 44* (1), 1–44.

Cremers, K., F. Gaessler, D. Harhoff, C. Helmers, and Y. Lefouili (2016). Invalid but infringed? an analysis of the bifurcated patent litigation system. *Journal of Economic Behavior & Organization 131*, 218–242.

de Rassenfosse, G., W. E. Griffiths, A. B. Jaffe, and E. Webster (2021). Low-quality patents in the eye of the beholder: Evidence from multiple examiners. *The Journal of Law, Economics, & Organization 37* (3), 607–636.

Demougin, D. and C. Fluet (2008). Rules of proof, courts, and incentives. *The RAND Journal of Economics 39* (1), 20–40.

Devlin, A. (2008). Revisiting the presumption of patent validity. *Southwestern University Law Review 37* (2), 323–370.

Farmer, A. and P. Pecorino (1999). Legal expenditure as a rent-seeking game. *Public Choice 100* (3-4), 271–288.

Farrell, J. and C. Shapiro (2008). How strong are weak patents? *American Economic Review 98* (4), 1347–1369.

Friehe, T. and A. Wohlschlegel (2019). Rent seeking and bias in appeals systems. *The Journal of Legal Studies 48* (1), 117–157.

Ganglmair, B., C. Helmers, and B. J. Love (2022). The effect of patent litigation insurance: Theory and evidence from NPEs. *The Journal of Law, Economics, & Organization 38* (3), 741–773.

Graham, S. J., B. H. Hall, D. Harhoff, and D. C. Mowery (2002). Post-issue patent quality control: A comparative study of US patent Re-examinations and European patent oppositions. Technical report, National Bureau of Economic Research.

Graham, S. J. and N. Van Zeebroeck (2013). Comparing patent litigation across Europe: A first look. *Stanford Technology Law Review 17*(2), 655–708.

Guerra, A., B. Luppi, and F. Parisi (2019). Standards of Proof and Civil Litigation: A Game-Theoretic Analysis. *The B.E. Journal of Theoretical Economics 19*(1), 20170005.

Guerra, A., B. Luppi, and F. Parisi (2022). Do presumptions of negligence incentivize optimal precautions? *European Journal of Law and Economics 54*, 349–368.

Helmers, C. and B. J. Love (2023). Patent Validity and Litigation: Evidence from US Inter Partes Review. *The Journal of Law and Economics 66*(1), 53–81.

Henkel, J. and H. Zischka (2019). How many patents are truly valid? Extent, causes, and remedies for latent patent invalidity. *European Journal of Law and Economics 48*(2), 195–239.

Hirshleifer, J. and E. Osborne (2001). Truth, effort, and the legal battle. *Public Choice 108*(1-2), 169–195.

Kesan, J. P. and A. A. Gallo (2006). Why bad patents survive in the market and how should we change - The private and social costs of patents. *Emory Law Journal 55*(1), 61–140.

Klimczak, R. R. (2012). I4i and the Presumption of Validity: Limited Concerns over the Insulation of Weak Patents. *Berkeley Technology Law Journal 27*(Annual Review), 299–324.

Konrad, K. A. (2000). Sabotage in rent-seeking contests. *The Journal of Law, Economics, & Organization 16*(1), 155–165.

Konrad, K. A. (2009). *Strategy and Dynamics in Contests.* New York, NY: Oxford University Press.

Krueger, A. O. (1974). The political economy of the rent-seeking society. *The American Economic Review 64*(3), 291–303.

Lagerlöf, J. (1997). Lobbying, information, and private and social welfare. *European Journal of Political Economy 13*(3), 615–637.

Lagerlöf, J. N. (2007). A theory of rent seeking with informational foundations. *Economics of Governance 8*(3), 197–218.

Lei, Z. and B. D. Wright (2017). Why weak patents? Testing the examiner ignorance hypothesis. *Journal of Public Economics 148*, 43–56.

Lemley, M. A. and C. Shapiro (2005). Probabilistic patents. *Journal of Economic Perspectives 19*(2), 75–98.

Massenot, B., M. Maraki, and C. Thöni (2021). Litigation Spending and Care under the English and American Rules: Experimental Evidence. *American Law and Economics Review 23*(1), 164–206.

Moore, K. A. (2002). Juries, patent cases & a lack of transparency. *Houston Law Review 39*(3), 779–802.

Ottoz, E. (2019). Patent litigation. In A. Marciano and G. B. Ramello (Eds.), *Encyclopedia of Law and Economics*, pp. 1559–1566. New York, NY: Springer.

Palangkaraya, A., E. Webster, and P. H. Jensen (2011). Misclassification between patent offices: Evidence from a matched sample of patent applications. *Review of Economics and Statistics 93*(3), 1063–1075.

Pénin, J. (2012). Strategic uses of patents in markets for technology: A story of fabless firms, brokers and trolls. *Journal of Economic Behavior & Organization 84*(2), 633–641.

Rai, A. K. and S. Vishnubhakat (2019). The USPTO's Patent Trial and Appeal Board. In B. Depoorter, P. Menell, and D. Schwartz (Eds.), *Research Handbook on the Economics of Intellectual Property Law*, pp. 92–103. Cheltenham, UK: Edward Elgar Publishing.

Sanchirico, C. W. (1997). The burden of proof in civil litigation: A simple model of mechanism design. *International Review of Law and Economics 17*(3), 431–447.

Schwartz, D. L. and C. B. Seaman (2013). Standards of proof in civil litigation: An experiment from patent law. *Harvard Journal of Law & Technology 26*(2), 429–480.

Seaman, C. B. (2019). Presumption of validity. In B. Depoorter, P. Menell, and D. Schwartz (Eds.), *Research Handbook on the Economics of Intellectual Property Law*, pp. 350–366. Cheltenham, UK: Edward Elgar Publishing.

Skaperdas, S. and S. Vaidya (2012). Persuasion as a contest. *Economic Theory 51*(2), 465–486.

The Economist (2015, August). A question of utility. `https://www.economist.com/international/2015/08/08/a-question-of-utility`.

Tullock, G. (1967). The welfare costs of tariffs, monopolies, and theft. *Western Economic Journal 5*(3), 224–32.

Tullock, G. (1978). *Trials on Trial: The Pure Theory of Legal Procedure*. New York, NY: Columbia University Press.

Tullock, G. (1997). The case against the common law. In C. K. Rowley (Ed.), *The Selected Works of Gordon Tullock*, Volume 9, pp. 397–455. Indianapolis, Indiana: Liberty Fund.

Zhang, J. and J. Zhou (2016). Information disclosure in contests: A bayesian persuasion approach. *The Economic Journal 126*(597), 2197–2217.