Supplementary materials for "Presumption of patent validity and litigation incentives"

Alice Guerra* Tapas Kundu†

Appendix B

This document contains supplemental materials for Guerra and Kundu, "Presumption of patent validity and litigation incentives." It has two sections. In Section B.1, we present proofs of the lemmas A.1, A.2, and A.3, used in the paper. In Section B.2, we discuss our modeling approach and its key assumption.

B.1 Additional proofs

Proof of Lemma A.1. Observe that

$$\frac{de^c}{d\theta} = \mu\Bigg[q_{h1}h^\mu \frac{d}{d\theta}\frac{\theta\,(1-\theta)}{((1-\theta)+\theta h^\mu)^2} + q_0\frac{d\theta\,(1-\theta)}{d\theta} + q_{1h}h^\mu \frac{d}{d\theta}\frac{\theta\,(1-\theta)}{((1-\theta)\,h^\mu+\theta)^2}\Bigg],$$

which, after simplifying, reduces to

$$\frac{de^c}{d\theta} = \mu\Bigg[\frac{q_{h1}h^\mu\,(h^\mu+1)}{((1-\theta)+\theta h^\mu)^3}\left(\frac{1}{h^\mu+1}-\theta\right) + 2q_0\left(\frac{1}{2}-\theta\right) + \frac{q_{1h}h^\mu\,(h^\mu+1)}{((1-\theta)\,h^\mu+\theta)^3}\left(\frac{h^\mu}{h^\mu+1}-\theta\right)\Bigg].$$

*Affiliation: Department of Economics, University of Bologna.
E-mail: alice.guerra3@unibo.it. Address: via Angherà 22, 47921 Rimini, Italy. Phone: +39 0541 434 266. ORCID: 0000-0003-1956-0270.
†Affiliation: Oslo Business School, Oslo Metropolitan University.
Email: tapas.kundu@oslomet.no. Address: Pilestredet 35, 0166 Oslo, Norway. Phone: +47 67 23 50 47. ORCID: 0000-0003-1814-7230.

Because $0 < 1/(h^\mu + 1) < 1/2 < h^\mu/(h^\mu + 1) < 1$, $(de^c/d\theta)$ is strictly positive for all $\theta \leq 1/(h^\mu+1)$ and $(de^c/d\theta)$ is strictly negative for all $\theta \geq h^\mu/(h^\mu+1)$, these observations together prove the first part of the lemma. Further, it follows that the global maximum lies in $[1/(h^\mu + 1), h^\mu/(h^\mu + 1)]$ and multiple local optima might exist for $\theta \in [1/(h^\mu + 1), h^\mu/(h^\mu + 1)]$, depending on the curvature of e^c. To study the curvature, we examine the second-order derivative.

$$\begin{aligned}\frac{d^2e^c}{d\theta^2} = \mu\Bigg[&q_{h1}h^\mu\frac{d}{d\theta}\frac{1-\theta\left(h^\mu+1\right)}{\left(\left(1-\theta\right)+\theta h^\mu\right)^3} + q_0\frac{d\left(1-2\theta\right)}{d\theta}\\ &+ q_{1h}h^\mu\frac{d}{d\theta}\frac{h^\mu-\theta\left(h^\mu+1\right)}{\left(\left(1-\theta\right)h^\mu+\theta\right)^3}\Bigg].\end{aligned}$$

which, after simplifying, reduces to

$$\begin{aligned}\frac{d^2e^c}{d\theta^2} = \mu\Bigg[&\frac{2q_{h1}h^\mu\left(h^{2\mu}-1\right)}{\left(\left(1-\theta\right)+\theta h^\mu\right)^4}\left(\theta-\frac{2h^\mu-1}{h^{2\mu}-1}\right) - 2q_0\\ &+ \frac{2q_{1h}h^\mu\left(h^{2\mu}-1\right)}{\left(\left(1-\theta\right)h^\mu+\theta\right)^4}\left(\frac{h^\mu\left(h^\mu-2\right)}{h^{2\mu}-1}-\theta\right)\Bigg].\end{aligned}$$

Observe that if $h^\mu \leq 2$, then $(2h^\mu - 1)/(h^{2\mu} - 1) \geq 1$ and $h^\mu(h^\mu - 2)/(h^{2\mu} - 1) \leq 0$, which together imply $(d^2e^c/d\theta^2)$ is negative, or equivalently, e^c is globally concave for $\theta \in [0, 1]$. □

Proof. [**Proof of Lemma A.2**]Observe that

$$\begin{aligned}F\left(\theta\right)-k =&\pi^e-\left(1-e^c\right) = e^c-\left(1-\pi^e\right)\\ =&\Bigg[q_{h1}\left(\frac{\theta\left(1-\theta\right)\mu h^\mu}{\left(\left(1-\theta\right)+\theta h^\mu\right)^2}-\frac{\left(1-\theta\right)}{\left(1-\theta\right)+\theta h^\mu}\right)\\ &+q_0\left(\theta\left(1-\theta\right)\mu-\left(1-\theta\right)\right)+q_{1h}\left(\frac{\theta\left(1-\theta\right)\mu h^\mu}{\left(\left(1-\theta\right)h^\mu+\theta\right)^2}-\frac{\left(1-\theta\right)h^\mu}{\left(1-\theta\right)h^\mu+\theta}\right)\Bigg]\\ =&\Bigg[-\frac{q_{h1}\left(1-\theta\right)\left(\left(1-\theta\right)+h^\mu\theta\left(1-\mu\right)\right)}{\left(\left(1-\theta\right)+\theta h^\mu\right)^2}-q_0\left(1-\theta\right)\left(1-\theta\mu\right)\\ &-\frac{q_{1h}h^\mu\left(1-\theta\right)\left(h^\mu\left(1-\theta\right)+\theta\left(1-\mu\right)\right)}{\left(\left(1-\theta\right)h^\mu+\theta\right)^2}\Bigg].\end{aligned}$$

The first-order derivatives of the three components of are as follows:

$$
\begin{aligned}
(i)\ & \frac{d}{d\theta}\left(-\frac{q_{h1}\left(1-\theta\right)\left(\left(1-\theta\right)+h^{\mu}\theta\left(1-\mu\right)\right)}{\left(\left(1-\theta\right)+\theta h^{\mu}\right)^{2}}\right) \\
&=\frac{q_{h1}}{\left(\left(1-\theta\right)+\theta h^{\mu}\right)^{3}}\left[2h^{\mu}\left(1-\theta\right)\mu+h^{\mu}\left(1-\mu\right)\left(\left(1-\theta\right)+\theta h^{\mu}\right)\right];\\
(ii)\ & \frac{d}{d\theta}\left(-q_{0}\left(1-\theta\right)\left(1-\theta\mu\right)\right)=q_{0}\left(\left(1-\theta\mu\right)+\mu\left(1-\theta\right)\right);\\
(iii)\ & \frac{d}{d\theta}\left(-\frac{q_{1h}h^{\mu}\left(1-\theta\right)\left(h^{\mu}\left(1-\theta\right)+\theta\left(1-\mu\right)\right)}{\left(\left(1-\theta\right)h^{\mu}+\theta\right)^{2}}\right)\\
&=\frac{q_{1h}}{\left(\left(1-\theta\right)h^{\mu}+\theta\right)^{3}}\Bigg[2\left(h^{\mu}-1\right)\theta\left(1-\theta\right)\mu\\
&\qquad+\left(\left(1-\theta\mu\right)+\mu\left(1-\theta\right)\right)\left(\left(1-\theta\right)h^{\mu}+\theta\right)\Bigg];
\end{aligned}
$$

Each of these derivatives are strictly positive when $0<\mu\leq 1$. Therefore, $F\left(\theta\right)+k$, and equivalently, $F\left(\theta\right)$ is strictly increasing in $\theta\in[0,1]$. □

Proof of Lemma A.3. Observe that

$$
\begin{aligned}
G\left(\theta\right)+k &= q_{h1}\left(\frac{\theta h^{\mu}}{\left(1-\theta\right)+\theta h^{\mu}}-\frac{\theta\left(1-\theta\right)\mu h^{\mu}}{\left(\left(1-\theta\right)+\theta h^{\mu}\right)^{2}}\right)+q_{0}\left(\theta-\theta\left(1-\theta\right)\mu\right)\\
&\quad+q_{1h}\left(\frac{\theta}{\left(1-\theta\right)h^{\mu}+\theta}-\frac{\theta\left(1-\theta\right)\mu h^{\mu}}{\left(\left(1-\theta\right)h^{\mu}+\theta\right)^{2}}\right)\\
&=\frac{q_{h1}h^{\mu}\theta\left(h^{\mu}\theta+\left(1-\mu\right)\left(1-\theta\right)\right)}{\left(\left(1-\theta\right)+\theta h^{\mu}\right)^{2}}+q_{0}\left(\theta-\theta\left(1-\theta\right)\mu\right)\\
&\quad+\frac{q_{1h}\theta\left(\theta+h^{\mu}\left(1-\mu\right)\left(1-\theta\right)\right)}{\left(\left(1-\theta\right)h^{\mu}+\theta\right)^{2}}.
\end{aligned}
$$

The first-order derivatives of the three components of are as follows:

$$
\begin{aligned}
(i)\ & \frac{d}{d\theta}\frac{q_{h1}h^{\mu}\theta\left(h^{\mu}\theta+(1-\mu)(1-\theta)\right)}{\left((1-\theta)+\theta h^{\mu}\right)^{2}} \\
&= \frac{q_{h1}h^{\mu}}{\left((1-\theta)+\theta h^{\mu}\right)^{3}}\Big[2\theta\mu\left(h^{\mu}-1\right)(1-\theta) \\
&\qquad + \left(2\theta\mu+(1-\mu)\right)\left((1-\theta)+\theta h^{\mu}\right)\Big]; \\
(ii)\ & \frac{d}{d\theta}q_{0}\left(\theta-\theta(1-\theta)\mu\right)=q_{0}\left(2\theta\mu+(1-\mu)\right); \\
(iii)\ & \frac{d}{d\theta}\frac{q_{1h}\theta\left(\theta+h^{\mu}(1-\mu)(1-\theta)\right)}{\left((1-\theta)h^{\mu}+\theta\right)^{2}} \\
&= \frac{q_{1h}}{\left((1-\theta)h^{\mu}+\theta\right)^{3}}\left[2\theta\mu+h^{\mu}(1-\mu)\left((1-\theta)h^{\mu}+\theta\right)\right];
\end{aligned}
$$

Each of these derivatives are strictly positive when $0<\mu\leq 1$. Therefore, $G(\theta)+k$, and equivalently, $G(\theta)$ is strictly increasing in $\theta\in[0,1]$. □

B.2 Discussion of the modeling approach and assumptions

In this section, we discuss our modeling approach and some key assumption.

Probabilistic decision rule: We consider a probabilistic decision rule to determine the outcome of a litigation contest. An alternative approach is to utilize a deterministic decision rule, similar to those employed in all-pay auction models within the contest literature. For example, P wins if the difference or ratio between the evidence presented by the two parties exceeds a certain threshold.[1] However, this intuitive form of decision rule does come with its drawbacks. The equilibrium of the game typically only exists in mixed strategies. Working with mixed-strategy equilibria is—or at least, appears to us as less appealing in our research, for two reasons. First, in addition to the uncertainty surrounding the true state, there are two additional sources of randomness: one from the equilibrium strategy and the other from the stochastic evidence-production function. These two sources will interact to produce evidence in equilibrium. This interaction complicates the process of updating beliefs based on the observed evidence profile, making

[1]See Konrad (2002), Kirkegaard (2012) for examples of all-pay auction contests, and Konrad (2009) for a discussion of the literature.

the subsequent analysis quite intractable.

Further, the interpretation of a mixed strategy equilibrium poses particular challenges in the context of litigation. A party adheres to a mixed strategy in equilibrium only if it receives the same payoff from every pure action over which it chooses to randomize. In our case, this implies, for example, that the plaintiff is indifferent among a set of expenses she would incur. However, if the plaintiff, instead of playing stochastically, consistently opts for a specific cost level based on her preferences, she would still achieve the same return but would impose an externality on the other player, ultimately disrupting the equilibrium consensus. The consensus relies on the understanding that it is preferable to introduce randomness since no pure action is strictly preferred.[2] In practice, it is challenging to envision how legal firms, which invest substantial time and resources in carefully evaluating every aspect of a case, would agree to embrace a mixed strategy concerning expenses in the first place.

Modeling the presumption bias: While legal scholarship has always regarded the presumption as a source of asymmetry, it remains unclear how this asymmetry is imposed. We approach the asymmetry in terms of a biased belief, where the judge evaluates evidence under the influence of this biased belief. Alternatively, one could consider the possibility that the judge begins with an unbiased belief but examines the evidence in a biased manner. Our motivation for adopting the former approach stems from the observation that pre-litigation activities such as the patent granting procedure may influence the judge to rule in favor of the patent holder. As Moore (2002) highlights, "practitioners and scholars alike have frequently opined that juries are not likely to invalidate patents because juries favor inventors and are unlikely to second-guess the Patent Office that has technically trained examiners who already issued the patents." Thus, the bias may be imposed even before the examination of evidence, and the parties' decisions on resource spending are influenced by this bias within the due process.

However, it is possible to model the presumption by introducing asymmetry in other ways. For instance, Skaperdas and Vaidya (2012) considers an alternative version of the

[2]This line of critique is also acknowledged in the literature; see Tullock (1985).

evidence-based persuasion framework where asymmetry is introduced in the decision-making rule: Party P wins if the posterior probability surpasses an exogenous threshold, which can be interpreted as the degree of bias. Under common knowledge, the framework reduces to an all-pay auction model, resulting in equilibrium in mixed strategies. As mentioned previously, employing mixed strategies significantly reduces analytical tractability in our framework.

In our model, the bias appears in the following form: bias $= (\theta - m) = \alpha\,(1-m)$. The bias is decreasing in the underlying merit of the patent—a high-merit patent would be less affected by the presumption criterion. An alternative modeling assumption could be considering a constant bias at every level of a patent's merit. However, given the natural bounds of probabilities, the assumption of constant bias is less appealing. Furthermore, we expect the presumption to have a positive influence on the court's belief in favor of patent validity, going beyond the true merit of the patent. In simpler terms, under PV, $\theta - m$ should be positive. Expressing the bias as a specific function of α restricts the generalizability of our analysis. Nonetheless, we have used α in an ad hoc manner: it is necessary to derive a closed-form solution of the litigation contest. However, our analysis of the effect of PV does not explicitly consider derivatives with respect to α.

Interpretation of evidence: In the model, our interpretation of evidence focuses on the intensity of evidence rather than its specific contents. By doing so, we abstract away several crucial aspects of evidence collection, such as the strategies employed by legal teams in pursuing different lines of inquiry and the strategic considerations involved in choosing one aspect over another. While analyzing these concepts can offer additional insights into the strategic decisions related to evidence collection, we maintain the tractability of our framework by representing intensity as a single parameter in our model.

Sensitivity to the evidence: In our model, the parameter $0 < \mu \leq 1$ measures sensitivity to the evidence. We impose an upper bound on to ensure existence of the Nash equilibrium. Alternative interpretations of the effects of μcan be found in Hirshleifer (1989) and Jia (2008). Following Hirshleifer (1989), μ can be interpreted as an index of the mass effect in the contest. It can be shown that the quasi-concavity of the CSF imposes an upper bound on the value of μ. Jia (2008) interprets this parameter as a measure of noise. As μ approaches zero, evidence would have little influence, and the posterior would be similar to the prior. Conversely, as μ approaches infinity, the contest outcome is almost determined by an all-pay auction, and the party with a higher intensity of evidence is guaranteed to win. Nevertheless, for large values of μ, the characterization of equilibrium in contest with non-anonymous CSF remains an unsolved issue.

Measuring judgment error: Previous studies have often assigned exogenous weights to these two types of errors, a false positive and a false negative. In civil offenses, the two errors are commonly treated equally, whereas in criminal offenses, a false positive case is considered as more serious (Burtis et al., 2017; Clermont and Sherwin, 2002). However, by focusing on the probability of committing any error, we effectively make these weights endogenous. For example, a false positive case can only occur when the state is invalid. Consequently, the probability of encountering a false positive case is naturally higher for low-merit patents, which are more likely to be invalid. Additionally, assigning pre-fixed, unequal weights to the two errors across patents of all merits would introduce an additional source of bias in favor of one party, making it more challenging to disentangle the effect of the biased prior.

Assumption 1: This assumption ensures concavity of the resource spending level, which allows us to study comparative static effects on resource dissipation and judgment error in a tractable form. The assumption is not necessary for characterizing the equilibrium regimes or for determining the effect of shifting the prior on the equilibrium regime. Relaxing this assumption would impact the uniqueness of θ^R. If the assumption is violated, e^c is still increasing for $\theta \leq 1/\left(1 + h^\mu(\mu - 1)\right)$ and is decreasing for $\theta \geq h^\mu/\left(h^\mu + \mu - 1\right)$; however, there could be multiple local maxima for the intermediate values of θ. Therefore, in the absence of Assumption 1, we will still observe that the presumption is going to increase (decrease) the aggregate resource dissipation for sufficiently low-merit (high-merit) patents; however, the uniqueness of the threshold on the θ-values will not hold.

Non-bifurcated legal system: The differentiation between bifurcated and non-bifurcated systems has significant implications for defense strategies, as pointed out by Cremers et al. (2016). Cremers et al. (2016) note that asserting invalidity is a common defense strategy in non-bifurcated systems. Although costlier, in bifurcated systems, it is not uncommon for defendants to initiate parallel invalidity proceedings. This approach sometimes leads to divergent outcomes: a patent may be initially deemed infringed but later invalidated in a separate proceeding (Cremers et al., 2016). The functioning of the US legal system is, instead, more complex: claims of non-infringement and invalidity are addressed simultaneously in US courts, whereas Inter Partes Review deals with invalidity proceedings separately. On infringement and validity proceedings, see also Langinier and Marcoul (2009); Krasteva et al. (2020).

Litigation's effect on market outcome: We assume that the outcome of the lawsuit will not affect the market outcome. This assumption clearly holds in NPE litigation, which account for the majority of patent infringement cases, especially in the technology sector (Riess, 2023; Chen et al., 2023). The presumption, however, applies to both PE and NPE litigation and the mechanism we modeled here would not necessarily be different between the two scenarios. Nonetheless, we chose not to discuss market competition in this paper for two reasons. Firstly, we aim to keep our model streamlined to concentrate on how the presumption criteria influence incentives for evidence collection during disputes. Although market structure can impact litigation incentives, it is not evident that its effects would interact with the impact of the presumption in a non-obvious manner. Second, market competition introduces additional sources of disparity between the litigating parties and have welfare implications, which are also not the focus of this paper.

Out-of-court settlement: Finally, our framework does not address the possibility of out-of-court settlement. A suitable out-of-court settlement offer can Pareto dominate the outcome of a litigation trial. This observation presents an interesting dilemma: why would litigation trials be chosen over settlements? The literature has uncovered various factors, including, among others, asymmetric information or asymmetric beliefs between disputing parties, and indivisibility of the prize; see Spier (2007) for a comprehensive survey.[3]

Our framework does not assume any of these factors, implying there is always potential for a settlement offer to improve outcomes for both sides compared to a trial. In our model, the defendant's only alternative to a trial is accepting a default judgment. We exclude pre-trial settlements to focus on analyzing how presumption bias influences the strategic pursuit of evidence. While we recognize the potential influence of presumption bias on settlement offers, a detailed examination of this aspect falls outside the scope of this paper and is left for future research.

References

Burtis, M. M., J. B. Gelbach, and B. H. Kobayashi (2017). Error costs, legal standards of proof, and statistical significance. *Supreme Court Economic Review 25*(1), 1–57.

Chen, F., Y. Hou, J. Qiu, and G. Richardson (2023). Chilling effects of patent trolls. *Research Policy 52*(3), 104702.

Clermont, K. M. and E. Sherwin (2002). A comparative view of standards of proof. *American Journal of Comparative Law 50*(2), 243–276.

Crampes, C. and C. Langinier (2002). Litigation and settlement in patent infringement cases. *RAND Journal of Economics 33*(2), 258–274.

Cremers, K., F. Gaessler, D. Harhoff, C. Helmers, and Y. Lefouili (2016). Invalid but infringed? an analysis of the bifurcated patent litigation system. *Journal of Economic Behavior & Organization 131*, 218–242.

Hirshleifer, J. (1989). Conflict and rent-seeking success functions: Ratio vs. difference models of relative success. *Public Choice 63*(2), 101–112.

Jia, H. (2008). A stochastic derivation of the ratio form of contest success functions. *Public Choice 135*(3/4), 125–130.

[3]See also Reinganum and Wilde (1986); Spier (1992) for analyses of settlement in litigation. Additionally, for analyses specific to patent litigation, see Meurer (1989); Crampes and Langinier (2002).

Kirkegaard, R. (2012). Favoritism in asymmetric contests: Head starts and handicaps. *Games and Economic Behavior 76*(1), 226–248.

Konrad, K. A. (2002). Investment in the absence of property rights; the role of incumbency advantages. *European Economic Review 46*(8), 1521–1537.

Konrad, K. A. (2009). *Strategy and Dynamics in Contests*. New York, NY: Oxford University Press.

Krasteva, S., P. Sharma, and C. Wang (2020). Patent policy, imitation incentives, and the rate of cumulative innovation. *Journal of Economic Behavior & Organization 178*, 509–533.

Langinier, C. and P. Marcoul (2009). Contributory infringement rule and patents. *Journal of Economic Behavior & Organization 70*(1), 296–310.

Meurer, M. J. (1989). The settlement of patent litigation. *The RAND Journal of Economics 20*(1), 77–91.

Moore, K. A. (2002). Juries, patent cases & a lack of transparency. *Houston Law Review 39*(3), 779–802.

Reinganum, J. F. and L. L. Wilde (1986). Settlement, litigation, and the allocation of litigation costs. *The RAND Journal of Economics 17*(4), 557–566.

Riess, D. (2023). Small business vs. patent trolls. *Strategic Finance 105*(6), 32–37.

Skaperdas, S. and S. Vaidya (2012). Persuasion as a contest. *Economic Theory 51*(2), 465–486.

Spier, K. E. (1992). The dynamics of pretrial negotiation. *The Review of Economic Studies 59*(1), 93–108.

Spier, K. E. (2007). Litigation. *Handbook of law and economics 1*, 259–342.

Tullock, G. (1985). Back to the bog: Efficient rents 3. *Public Choice 46*(3), 259–263.