Presumption of patent validity and litigation incentives

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Abstract

We analyze the effects of the presumption of patent validity on litigation incentives. We consider a litigation game between a patent holder and an alleged infringing firm. A court resolves the dispute if there is a trial. We model the court's decision-making as a learning process based on evidence and consider the presumption as a factor influencing the court's prior belief of patent validity. The presumption affects the trial outcome in two ways—by biasing the prior, and by affecting the incentives to invest in evidence-seeking activities. We find that the presumption can affect the incidence of trials in either direction. We further reveal its countervailing efficiency effects—likely raising the error of judgment, but decreasing the resource dissipation—when there is high uncertainty about the patent's objective merit. In these contexts, our findings suggest a cautious or limited application of the presumption.

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1 Introduction

Granted patents generally enjoy a presumption of validity in court proceedings (Seaman, 2019). The burden of proof falls upon the infringer, who has to provide relevant facts about patent invalidity and convince the court with clear and convincing evidence, which is a higher standard of proof than the "preponderance of evidence" typically required in civil lawsuits.[1] In common law

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[1] To clarify some of the terminology that will be used in our analysis, the standard of proof is the level of certainty and the amount of evidence necessary to prove a claim in a trial; the burden of proof identifies the party who must offer evidence to raise a claim in litigation (Adler and Michael, 1931; Sanchirico, 1997). These concepts are practically interdependent but theoretically distinct (Davis, 1994; Clermont and Sherwin, 2002; Schwartz and Seaman, 2013; Talley, 2013; Guerra et al., 2019b; 2022).

countries, by statute "a patent shall be presumed valid" and the "burden of establishing invalidity of a patent or any claim thereof shall rest on the party asserting such invalidity" (35 USC ∮282, 1952).[2] A similar presumption holds for European patents and Supplementary Protection Certificates (Graham et al., 2002; Seaman, 2019).

There are different theoretical justifications in support of the presumption of [patent] validity—the most common one being deference to the patent office's expertise in evaluating patent applications, along with its agency flexibility and political accountability (Lichtman and Lemley, 2007; Devlin, 2008; Seaman, 2019; Narechania, 2021).[3] Notwithstanding this general rationale, the application of the presumption is not uniform neither across jurisdictions, nor across legal proceedings within a jurisdiction (Ottoz, 2019). Differences across national jurisdictions are substantial, especially within Europe (Graham et al., 2002; Luginbuehl, 2011; Graham and Van Zeebroeck, 2013; Cremers et al. 2017). For example, the Netherlands apply no presumption, whereas Sweden and Denmark generally apply a strong presumption of validity unless there is clear evidence of invalidity. Recent case laws have also considered a reversed presumption, namely a presumption of invalidity (e.g., *Syral Belgium v. Roquette Frères).*[4]

Differences across legal proceedings within a jurisdiction are substantial as well. In the U.S., the presumption of validity is applied in court litigation, whereas no presumption or a "weakened" presumption is applied in post-issuance administrative proceedings before the U.S. Patent and Trademark Office's (USPTO) Patent Trial and Appeal Board (PTAB). These proceedings—through which the public may ask the patent office to reassess the validity of granted patents—include inter partes review, post-grant review, and ex parte re-examination (Rai and Vishnubhakat, 2019; Helmers and Love, 2023).[5] For example, in the inter partes review—which has been introduced by the America Invents Act on September 16, 2012—a petitioner can challenge the validity of a U.S. patent under a "preponderance of evidence" standard, showing that claims are more likely unpatentable than not (Helmers and Love, 2023).[6] This significantly reduces the standard of proof,

[2]The Congress amended Section 282 in 1965, 1975, 1995, and 2011. The sentences: "A patent shall be presumed valid," and "The burden of establishing invalidity of a patent or any claim thereof shall rest on the party asserting such invalidity," remained in the statute. See, e.g., *Microsoft Corp. v. i4i Limited Partnership*, 131 S.Ct. 2238 (2011).

[3]See also *Cuozzo Speed Techs. v. Lee*, 136 S. Ct. 2131, 2144 (2016).

[4]*Syral Belgium v. Roquette Frères* (Supreme Court, Belgium, 12 September 2014, Case No. C.13.0232) challenged the *prima facie* validity of a patent based on a decision from other European jurisdictions (UK and France) invalidating the patent. In this case, a foreign legal decision may generate a presumption of invalidity for the Belgian part of the patent.

[5]Regarding inter partes review, "In an inter partes review [...], the petitioner shall have the burden of proving a proposition of unpatentability by a preponderance of the evidence." 35 U.S.C. ∮ 316(e). See also *In re* Global Tel*link Corp., IPR 2014-00493, 2014 WL 4715524, Sept. 17, 2014 ("There is no presumption of validity as to the challenged claims in an inter partes review."). Regarding ex parte re-examination, "the standard of proof – a preponderance of evidence – is substantially lower than in a civil case [and] there is no presumption of validity." (*In re* Swanson, 540 F.3d 1368, 1377, Fed. Cir. 2008).

[6]35 U.S.C. § 316(e) (inter partes review), 326(e) (post-grant review). See also the Trial Practice and Procedure Rules, confirming that "the default evidentiary standard is a preponderance of the evidence" (37 CFR § 42.1(d)). For

compared to patent litigation in court where the "clear and convincing evidence" standard is applied. Consequently, an infringement defendant may prefer to challenge patent validity in post-issuance administrative proceedings, rather than in or in parallel with litigation in district courts (Love and Ambwani, 2014; Rai and Vishnubhakat, 2019; Seaman, 2019; Helmers and Love, 2023).

Beyond the discrepancies in the application of the presumption of validity across jurisdictions and legal proceedings, several critiques have been raised upon the mere presence of the presumption. Some scholars argued that the presumption should be abolished because it is not simply a procedural device, but rather a powerful mechanism for injecting pro-patentee bias (Lemley, 2000; Bohrer, 2004; Bock, 2014). An explicit statement in jury instructions that a patent is presumed valid makes juries less likely to invalidate patents (Moore, 2002). This is not problematic if the patent was correctly granted: in this case, the presumption protects the patent holder from any frivolous (or, non-meritorious) infringement litigation (Lichtman and Lemley, 2007; Seaman, 2019). The problem arises when patent offices make evaluation errors and grant patents that, on their merits, should not have been issued in the first place (de Rassenfosse et al., 2021).[7] These patents are referred to in the literature as "latently invalid" or "incorrectly granted" patents (Henkel and Zischka, 2019), and also as "bad" or "weak" patents (Kesan and Gallo, 2006; Choi and Gerlach, 2015; Lei and Wright, 2017).

The problem of latently invalid patents is sizable, especially in the U.S. (Federal Trade Commission, 2003; Levin, 2004; Lemley and Shapiro, 2005; The Economist, 2015).[8] Judicial review is generally one solution to correct latently invalid patents. Yet, only a small fraction of all patents is litigated—a mere 0.1%, as noted by Lemley and Shapiro (2005)— because of both the expensive district court litigation process and the presence of the presumption of validity (DiMatteo, 2002; American Intellectual Property Law Association, 2013; Helmers and Love, 2023).

Rather than entering litigation, innovators may find it more convenient to pay licensing and transaction costs of bargaining to reach a private agreement with the patentee, thereby leaving the latently invalid patent in the market (Kesan and Gallo, 2006). This exponentially increases the uncertainties of the innovation process by exposing innovators to the risk of non-meritorious liti-

more details about the inter partes review process, see, e.g., Helmers and Love (2023).

[7]Patent evaluation errors by patent offices as the USPTO may occur for different reasons, including low accuracy levels of reviewing patent applications (Quillen and Webster, 2001), political influence (Davis, 2004), inefficient incentives to grant valid patents (e.g., absence of penalties for incorrectly issued patents; Kesan, 2002). Evaluation errors are particularly common in high technology sectors, which are not areas of traditional patenting, and therefore it is more difficult for the patent office to gather prior art information (Kesan and Gallo, 2006). On the risk of errors by both patent examiners at patent offices and judges at civil court, see Buzzacchi and Scellato (2008).

[8]The problem is also present in Europe, but less pronounced (Fusco, 2013; Cremers et al., 2017; Henkel and Zischka, 2019). For example, Cremers et al. (2017) reported that about 30% of appealed patent suits have their initial decision overturned; Henkel and Zischka (2019) found a 75% invalidity rate of appeals at the German Federal Patent Court between 2000 and 2012. See also Palangkaraya et al. (2011), which analyzed patent applications granted by the USPTO and examined at both the European Patent Office and Japanese Patent Office during the 1990s. Their estimates reveal that 9.8% of patents were incorrectly granted.

gation (Farrell and Merges, 2004; Farrell and Shapiro, 2008; de Rassenfosse et al., 2021). A major driver of non-meritorious patent infringement litigation is represented by non-practicing entities (NPE; also called "patent trolls")—that is, patent holders whose sole purpose is to enforce patent rights to threaten litigation and demand licensing fees, rather than producing or selling products or services (Pénin, 2012; Haus and Juranek, 2018; Ganglmair et al., 2022).[9] The presumption of validity may have the double-edged effect of leveraging NPEs' litigation tactics and their abilities to extract licensing payments from producing firms (Patent Quality Improvement Hearings, 2003, p. 4).

The debate upon the presumption has not been limited to scholarly contributions. During court litigation cases, judges and juries discussed the application of the presumption and the level of validity which should be applied to issued patents (for a review, see Klimczak, 2012). For example, the Supreme Court articulated and confirmed the application of the presumption at common law in several legal cases—one of the most cited being *Microsoft Corp. v. i4i Ltd. Partnership.*[10] In other legal cases, the Supreme Court stated that the presumption can be weakened or even eliminated when the patent office did not or was not able to consider relevant prior art in its review of patent applications.[11]

The legal debates on the presumption, along with the discrepancies in its application across jurisdictions and legal proceedings, circle back to a basic, yet fundamental question: What are the effects of the presumption on litigation incentives? To our knowledge, no prior studies thus far formally addressed this question. This gap in the literature is problematic, especially considering that legal and empirical studies have suggested the important role of legal presumptions in patent litigation (Chatlynne, 2009; Chatlynne, 2010; Schwartz and Seaman, 2013; Bock, 2014; Henkel and Zischka, 2019; Seaman, 2019),[12] and some scholars and governmental institutions have repeatedly advocated reforms to overcome the presumption in court proceedings (Lichtman and Lemley, 2007; Daniel, 2008; Devlin, 2008; Alsup, 2009; Ottoz, 2019; Seaman, 2019; Gugliuzza, 2021).

[9]A classic example of a legal case involving a patent troll is *NTP, inc. v. Research in Motion*, 397 F. Supp. 2d 785 (E.D. Va. 2005), where a NTP (a Virginia-based patent holding company) sued Research in Motion (RIM, the manufacturers of Blackberry) for patent infringement over its Blackberry devices. Even if the patent office found that three disputed patents should have not been granted in the first place, the NTP won an out-of-court settlement of US$ 612 million from RIM. For a discussion of this and other patent litigation cases involving patent trolls, see, e.g., Chan and Fawcett (2005).

[10]*Microsoft Corp. v. i4i Ltd. Partnership*, 131 S. Ct. 2238, 564 U.S. 91, 180 L. Ed. 2d 131 (2011). For an extensive discussion on this legal case, see Schwartz and Seaman (2013). See also *Radio Corp. of America v. Radio Engineering Laboratories, Inc.*, 293 U.S. 1, 55 S. Ct. 928, 79 L. Ed. 163 (1934); *Austin Machinery Co. v. Buckeye Traction Ditcher Co.*, 13 F.2d 697 (6th Cir. 1926); *Coffin v. Ogden*, 85 U.S. 120, 21 L. Ed. 821 (1874); *Sciele Pharma Inc. v. Lupin Ltd.*, 684 F.3d 1253 (Fed. Cir. 2012).

[11]See, e.g., *Manufacturing Research Corp. v. Graybar Elec. Co.*, 679 F.2d 1355 (11th Cir. 1982); *Heyl & Patterson, Incorporated v. McDowell Company*, 317 F.2d 719 (4th Cir. 1963).

[12]Some theoretical contributions analyzed the effects of legal presumptions on individuals' choices in other contexts (e.g., in tort settings; Bernardo et al., 2000; Demougin and Fluet, 2008; Guerra et al., 2022).

In this article, we seek to fill this knowledge gap by providing an analytical framework that explains how the legal presumption affects litigation incentives and its efficiency implications. We present a tractable patent-litigation game between a patent holder and a potential infringing firm. A non-strategic decision-maker, for example, a judge or a jury in case of court proceedings, resolves the dispute if there is a trial. The game consists of three sequential stages: the patent holder's decision to sue; the infringing firm's decision to defend; the trial stage where parties invest resources to gather and present evidence in order to persuade the court to make a favorable decision. The court's prior belief of patent validity is influenced by the presumption criterion, whereas the posterior belief takes the evidence produced during the litigation trial into account. The presumption affects the outcome of a trial in two ways—by biasing the prior, and by affecting the incentive to invest in evidence-seeking activities.

We characterize the competing parties' litigation decisions and their investment decisions that arise in equilibrium. We show that the effect of the presumption on the possibility of a litigation trial can be ambiguous. We further analyze its effect on two important features of a litigation trial—resource dissipation and error of judgment. Both resource dissipation and error of judgment are inefficient for the society. One of the key insights from our analysis is that the presumption has countervailing efficiency effects when there is high uncertainty about the patent's objective merit, e.g., in contexts where the examination of patent applications is complex and invalid patents are granted more frequently by the patent office (e.g., in high technology sectors).

The countervailing effects arise because of how the presumption influences the incentive to invest in evidence-seeking activities. Specifically, when the decision-maker is highly uncertain about the patent's validity, the competing parties act aggressively. However, the presumption of validity biases the competition in favor of the patent holder. On the one hand, the bias dampens the intensity of competition, which in turn reduces resource dissipation—and this is, in general, socially beneficial. On the other hand, it also reduces parties' incentives to gather new evidence, which has a high positive learning effect especially when there is high uncertainty about the patent's merit.

Our model of the litigation game as a process of persuasion is inspired by the Bayesian approach applied to contests as in Skaperdas and Vaidya (2012). We model the litigation-trial stage following their basic contest setting with Bayesian learning, a probabilistic decision rule, and a stochastic evidence-production function.[13] However, our approach departs from theirs in several ways. First, our original analytical move is to introduce the presumption as a factor that influences the decision-maker's prior belief of patent validity, before examining any evidence. This approach helps to study how the tension between adjusting belief due to the presumption and that due to learning from evidence, affects the litigation incentives. In doing so, our article provides novel

[13] On modeling contest success functions in litigation contests, see, among many others, Hirshleifer and Osborne (2001); Farmer and Pecorino (1999).

extensions of contest theory to advance our understanding of litigation. Second, we consider a specific stochastic evidence-production function that gives closed-form solutions of the equilibrium strategies in terms of the primitives of our model. The stochastic nature of the evidence-production function brings the possibility of learning from evidence, and the closed-form solutions enable us to study the comparative static effects in a tractable manner. Last but not least, our focus is different from theirs. Skaperdas and Vaidya (2012) intend to provide an information-based foundation of the additive contest success function (CSF) in a litigation contest, whereas in our analysis, we consider the Bayesian inferential process at the trial stage as a building block, and study how a biased belief of the decision-maker arising from a legal presumption affects parties' incentives to arrive at a trial.

Our model shares common features with the contest framework used to analyze rent-seeking incentives in various applications in economics and political science, including conflict, lobbying, litigations, advertising, internal and external labor markets, and R&D competition; see, for example, Konrad (2000; 2009) and the references therein. In much of the existing works involving contests with fixed prizes, efforts are non-productive and purely rent-seeking in nature. In our framework, contest efforts, however, play a different role. Costly investment can lead to hard information that fosters a proper allocation of property rights. In this sense, we contribute to the informational lobbying literature, addressing issues of strategic information transmission for rent-seeking purposes. Formal models of strategic information provision in lobbying can be found in Austen-Smith and Wright (1992), Potters and Van Winden (1992), Bennedsen and Feldmann (2002; 2006), and Lagerlöf (1997; 2007). Related to our work is Lagerlöf (2007), in which multiple firms compete in a rent-seeking contest to win the monopoly right. These firms, based on their private but soft information, make costly investments to acquire hard information in order to persuade a regulator with the authority of granting a monopoly. Unlike Lagerlöf (2007), we do not consider the possibility of privately informed rent-seekers. Further, we follow a different line of inquiry. Lagerlöf (2007) studies the welfare implication of costly information acquisition, whereas our objective is to examine how a biased prior belief of the decision-maker affects information-seeking incentives and litigation incentives of the rent-seeking parties.

Our treatment of the information transmission process also differs from the recent but growing literature on Bayesian persuasion in contests (Zhang and Zhou, 2016; Feng and Lu, 2016; Clark and Kundu, 2021a; 2021b). These models of Bayesian persuasion typically consider a contest designer strategically committing to a signaling mechanism that could disclose hidden information, completely or partially, and a set of rent-seeking agents who make costly investment after updating their beliefs. Our approach is fundamentally different: in our model, the rent-seeking agents disclose hard information to persuade the uninformed decision-maker, who has little strategic interest over the nature of information provision.

Heterogeneity among players is commonly acknowledged as a significant factor limiting investments in contest settings (Chowdhury et al., 2023). The favored player can leverage its relative advantage to dissuade the opponent from making high investments, and can hence reduce their own investment while still maintaining a high probability of winning. There is a large literature exploring the role of heterogeneity in contests and documents the effectiveness of various policies—such as discrimination, affirmative action, head starts, and handicaps—in leveling the playing field to achieve competitive balance (for comprehensive reviews, see Mealem and Nitzan, 2016; Chowdhury et al., 2023). Our model share common features with this literature, particularly in considering the implications of the presumption bias on resource investment for evidence-seeking purposes. We focus on analyzing how the inherent asymmetry due to the presumption bias extends to litigation incentives and its potential impacts on judgment errors.

The rest of the article is organized as follows. Section 2 presents the model setup and assumptions. Section 3 characterizes the equilibrium. Section 4 analyzes the effects of the presumption on litigation incentives, resource dissipation, and error of judgment in the court's decision-making. Section 5 discusses some key features of our study, and concludes with suggestions for future research.

2 Model

2.1 A patent-litigation game

We consider a litigation game between a patent holder P and a potential infringing firm Q, both assumed to be risk neutral. The game proceeds in three stages.

- Stage 1: P decides whether to sue Q. If P does not sue, the game ends and both players receive their default payoffs, which are normalized to zero. Otherwise, the game moves to stage 2.

- Stage 2: Q decides whether to defend. If Q decides not to defend, she submits to a default judgment in which the court awards P damages without holding a trial. Without loss of generality, we normalize the damage compensation value to 1. If Q defends, the game moves to a trial at stage 3.

- Stage 3: If the game proceeds to the trial stage, then both parties incur a positive participation cost $k \in [0,1]$. In addition, the competing parties strategically spend costly resources to gather and present evidence favorable to their causes. Specifically, at the beginning of the trial, P and Q simultaneously choose the level of resources $e_P \geq 0$ and $e_Q \geq 0$, respectively. Based on the evidence and the legal environment, which reflects the merit of the patent and the presumption of its validity, the court determines whether the patent is deemed valid. We consider a probabilistic decision-making process, which we discuss in the following subsections. If the court finds the patent valid, Q pays P the damage compensation. Otherwise, she pays nothing. The game ends with the court's decision.

We normalize the expenditure function such that the costs of using resources of level e are also given by e.[14] The model assumes the American Rule regarding the payment of legal expenditures, with each party paying their own legal expenditures.[15]

There are two types of costs associated with a trial: a fixed participation cost and resources spent in collecting evidence. The participation costs involve costs, such as time spent and psychological stress, that parties must endure even if they opt not to allocate resources to gather evidence. In contrast, players strategically allocate their resources to improve their chances to secure a favorable outcome. In a default judgment, parties can avoid paying any trial-related costs.

We assume that the outcome of the lawsuit will not affect the market outcome. This assumption simplifies the payoff structure of the game—in our model, the parties are solely disputing over a transfer and they have equal valuation of the prize.[16]

2.2 Belief about the patent validity

Following Skaperdas and Vaidya (2012), we model the court's decision-making as a process of persuasion. The alleged infringing firm, Q, can defend itself by denying infringement, challenging the validity of the patent, or both. This scenario resembles a non-bifurcated legal system with an invalidity argument as a defense. In non-bifurcated systems, such as those in the UK or Italy, there is no separation between infringement and validity proceedings. Defendants can challenge a patent's validity within infringement proceedings. Conversely, in bifurcated legal systems like those in Germany and China, infringement and validity proceedings are handled separately in distinct courts (Cremers et al., 2016).[17]

[14]A convex cost function is sufficient for the existence of an interior solution to the payoff-maximization problem. However, having a specific functional form is beneficial for deriving a closed-form solution for the Nash equilibrium.

[15]With the English Rule, the litigation game may not have pure-strategy equilibrium for all parameter values. On the American vs English Rule, see, e.g., Massenot et al. (2021).

[16]The assumption clearly holds in NPE litigations, which account for the majority of patent infringement cases, especially in the technology sector (Riess, 2023; Chen et al., 2023). The presumption, however, applies to both PE and NPE litigations and the mechanism we modeled here would not necessarily be different between the two scenarios. Nonetheless, we chose not to discuss market competition in this paper for two reasons. Firstly, we aim to keep our model streamlined to concentrate on how the presumption criteria influence incentives for evidence collection during disputes. Although market structure can impact litigation incentives, it is not evident that its effects would interact with the impact of the presumption in a non-obvious manner. Second, market competition introduces additional sources of disparity between the litigating parties and have welfare implications, which are also not the focus of this paper.

[17]The differentiation between bifurcated and non-bifurcated systems has significant implications for defense strategies, as pointed out by Cremers et al. (2016). Cremers et al. (2016) note that asserting invalidity is a common defense

Our model focuses on an invalidity argument in a patent infringement litigation. Specifically, we develop a persuasion framework that focuses on seeking favorable evidence by both parties in relation to the question of validity. Assume that there are two possible states $s \in \{V, I\}$ of the world: one in which the patent is valid (V), and the other in which the patent is invalid (I). The court's prior, denoted by θ, and posterior, denoted by π, quantify its belief about the event $s = V$, before and after it considers the evidence produced during the trial, respectively.

The court's prior is a subjective assessment of the patent's validity before examining evidence. We assume that a patent's underlying merit and the presumption criterion influence the prior. The merit of a patent refers to the presumption-free belief about the state of patent's validity. We consider two presumption scenarios: one in which there is a presumption of validity (PV), and the other in which there is no presumption of validity (NP). We use a simple reduced-form representation of the prior of the following form:

$$\theta = \begin{cases} \alpha + (1-\alpha)m & \text{under PV} \\ m & \text{under NP} \end{cases}, \tag{1}$$

where $m \in (0,1)$ represents the merit and $\alpha \in (0,1)$ measures the relative weight on the presumption of validity. We can interpret the court's weighted prior under PV in the following way. A patent's validity can be assessed based on multiple criteria. One can find patent-specific free information on some criteria (of proportion $1-\alpha$) and the patent's intrinsic merit is useful in assessing validity on these grounds. On other criteria (of proportion α), no information is available and the presumption of patent's validity guides the prior assessment. It is worth noting that the effect of the presumption criterion is only limited to influencing the court's prior—it must not alter the competing parties' belief about the underlying state. Therefore, even when there is a presumption of validity, the competing parties' belief must be given by the presumption-free prior, i.e., $s = V$ with probability m.

The updating of the court's belief adheres to Bayes' Rule. Let E_i denote the evidence produced by $i \in \{P, Q\}$ during the trial. Given a prior θ, the court's posterior belief is given by

$$\begin{aligned} \pi &= \Pr(s = V \mid E_P, E_Q) \\ &= \frac{\Pr(E_P, E_Q \mid V)\Pr(s = V)}{\Pr(E_P, E_Q \mid I)\Pr(s = I) + \Pr(E_P, E_Q \mid V)\Pr(s = V)} = \frac{\theta L^V}{(1-\theta) + \theta L^V}, \end{aligned}$$

strategy in non-bifurcated systems. Although costlier, in bifurcated systems, it is not uncommon for defendants to initiate parallel invalidity proceedings. This approach sometimes leads to divergent outcomes: a patent may be initially deemed infringed but later invalidated in a separate proceeding (Cremers et al., 2016). The functioning of the US legal system is, instead, more complex: claims of non-infringement and invalidity are addressed simultaneously in US courts, whereas Inter Partes Review deals with invalidity proceedings separately. On infringement and validity proceedings, see also Langinier and Marcoul (2009); Krasteva et al. (2020).

where L^V is the likelihood ratio of patent validity based on evidence and is given by

$$L^V(E_P, E_Q) = \frac{\Pr(E_P, E_Q \mid V)}{\Pr(E_P, E_Q \mid I)}.$$

The likelihood ratio is also a subjective assessment of validity based on evidence. It is common in the literature to consider the likelihood ratio in power-law form (see, e.g., Skaperdas and Vaidya, 2012):

$$L^V(E_P, E_Q) = \left(\frac{E_P}{E_Q}\right)^{\mu},$$

where the pieces of evidence E_P and E_Q are expressed on the $(0, \infty)$ scale.[18] The parameter $0 < \mu \leq 1$ indicates sensitivity to the evidence.[19] We can therefore express the posterior probability as

$$\pi(E_P, E_Q, \theta) = \frac{\theta (E_P)^{\mu}}{(1-\theta)(E_Q)^{\mu} + \theta (E_P)^{\mu}}. \tag{2}$$

2.3 Production of evidence

We assume that the production of evidence depends on both the resources spent and the underlying state:

$$E_P(e_P, s) = \begin{cases} he_P & \text{with probability } f(s) \\ e_P & \text{with probability } 1 - f(s) \end{cases},$$
$$E_Q(e_Q, s) = \begin{cases} e_Q & \text{with probability } f(s) \\ he_Q & \text{with probability } 1 - f(s) \end{cases}, \tag{3}$$

where $h > 1$, $s \in \{V, I\}$, and

$$f(V) = 1 - f(I) = \gamma > \frac{1}{2}. \tag{4}$$

[18]Our interpretation of evidence focuses on the intensity of evidence rather than its specific contents. By doing so, we abstract away several crucial aspects of evidence collection, such as the strategies employed by legal teams in pursuing different lines of inquiry and the strategic considerations involved in choosing one aspect over another. While analyzing these concepts can offer additional insights into the strategic decisions related to evidence collection, we maintain the tractability of our framework by representing intensity as a single parameter in our model. Also see footnote 20 for an interpretation of the ratio form.

[19]We impose an upper bound on μ to ensure existence of the Nash equilibrium. Alternative interpretations of the effects of μ can be found in Hirshleifer (1989) and Jia (2008). Following Hirshleifer (1989), μ can be interpreted as an index of the "mass effect" in the contest. It can be shown that the quasi-concavity of the CSF imposes an upper bound on the value of μ. Jia (2008) interprets this parameter as a measure of noise. As μ approaches zero, evidence would have little influence, and the posterior would be similar to the prior. Conversely, as μ approaches infinity, the contest outcome is almost determined by an all-pay auction, and the party with a higher intensity of evidence is guaranteed to win. Nevertheless, for large values of μ, the characterization of equilibrium in contest with non-anonymous CSF remains an unsolved issue; see footnote 24 in Section 3.

This evidence production function reflects how the true state positively influences the availability of favorable evidence. For example, in state V, in which the patent is valid, P is more likely (with probability $\gamma > 1/2$) to produce a higher volume of favorable evidence compared to Q , given equal resource spending. The opposite effect is observed in state I. We assume that E_P and E_Q are independent variables. The court only observes the pair of evidence (E_P, E_Q) but does not observe the underlying resources P and Q spend.

2.4 A probabilistic decision rule

The court uses the following probabilistic decision rule to arrive at its verdict: Choose V with probability $\pi(E_P, E_Q, \theta)$ and choose I with probability $(1 - \pi(E_P, E_Q, \theta))$. The success probabilities of P and Q of winning the trial are therefore given by $\pi(E_P, E_Q, \theta)$ and $(1 - \pi(E_P, E_Q, \theta))$, respectively.[20] We let U_P and U_Q denote the expected payoffs of P and Q, respectively. We assume that all parameter values are common knowledge and analyze the Bayesian Nash equilibrium of the game.

2.5 Discussion of the modeling approach

In this subsection, we discuss some modeling assumptions we made for analytical tractability and intuitive interpretations of the results, and contrast them to alternative approaches.

To begin with, let us consider our use of a probabilistic decision rule to determine the outcome of a litigation contest. An alternative approach is to utilize a deterministic decision rule, similar to those employed in all-pay auction models within the contest literature. For example, P wins if the difference or ratio between the evidence presented by the two parties exceeds a certain threshold.[21] However, this intuitive form of decision rule does come with its drawbacks. The equilibrium of the game typically only exists in mixed strategies. Working with mixed-strategy equilibria is—or at least, appears to us as less appealing in our research, for two reasons. First, in addition to the uncertainty surrounding the true state, there are two additional sources of randomness: one from the equilibrium strategy and the other from the stochastic evidence-production function. These two

[20] We can express, after some algebraic manipulation, the ratio of winning probabilities as $\pi/(1-\pi) = L^V \cdot \theta/(1-\theta) = (E_P/E_Q)^\mu \cdot \theta/(1-\theta)$. This expression contains elements analogous to the litigation success function derived in Hirshleifer and Osborne (2001), exhibiting certain desirable properties. Following Hirshleifer and Osborne (2001), $\theta/(1-\theta)$ can be interpreted as the "fault factor," which is influenced by the subjective probabilities of the underlying state, at the pre-litigation stage. And, $(E_P/E_Q)^\mu$ can be interpreted as the "effort factor," which is influenced by the resources spent during the litigation process. Introducing evidence intensities in ratio form is justified by the fact that if the litigation trial results in a profile of evidence that does not provide any party with a special advantage, the outcome should depend solely on the fault factor. The direct effect of the presumption is limited to the fault factor, while its indirect effect on the effort factor arises from the litigating parties' strategic rent-seeking incentives.

[21] See Konrad (2002), Kirkegaard (2012) for examples of all-pay auction contests, and Konrad (2009) for a discussion of the literature.

sources will interact to produce evidence in equilibrium. This interaction complicates the process of updating beliefs based on the observed evidence profile, making the subsequent analysis quite intractable.

Further, the interpretation of a mixed strategy equilibrium poses particular challenges in the context of litigation. A party adheres to a mixed strategy in equilibrium only if it receives the same payoff from every pure action over which it chooses to randomize. In our case, this implies, for example, that the plaintiff is indifferent among a set of expenses she would incur. However, if the plaintiff, instead of playing stochastically, consistently opts for a specific cost level based on her preferences, she would still achieve the same return but would impose an externality on the other player, ultimately disrupting the equilibrium consensus. The consensus relies on the understanding that it is preferable to introduce randomness since no pure action is strictly preferred.[22] In practice, it is challenging to envision how legal firms, which invest substantial time and resources in carefully evaluating every aspect of a case, would agree to embrace a mixed strategy concerning expenses in the first place.

Next, let us consider our approach to modeling the presumption. While legal scholarship has always regarded the presumption as a source of asymmetry, it remains unclear how this asymmetry is imposed. We approach the asymmetry in terms of a biased belief, where the judge evaluates evidence under the influence of this biased belief. Alternatively, one could consider the possibility that the judge begins with an unbiased belief but examines the evidence in a biased manner. Our motivation for adopting the former approach stems from the observation that pre-litigation activities such as the patent granting procedure may influence the judge to rule in favor of the patent holder. As Moore (2002) highlights, "practitioners and scholars alike have frequently opined that juries are not likely to invalidate patents because juries favor inventors and are unlikely to second-guess the Patent Office that has technically trained examiners who already issued the patents." Thus, the bias may be imposed even before the examination of evidence, and the parties' decisions on resource spending are influenced by this bias within the due process.

However, it is possible to model the presumption by introducing asymmetry in other ways. For instance, Skaperdas and Vaidya (2012) considers an alternative version of the evidence-based persuasion framework where asymmetry is introduced in the decision-making rule: Party P wins if the posterior probability surpasses an exogenous threshold, which can be interpreted as the degree of bias. Under common knowledge, the framework reduces to an all-pay auction model, resulting in equilibrium in mixed strategies. As mentioned previously, employing mixed strategies significantly reduces analytical tractability in our framework.

[22] This line of critique is also acknowledged in the literature; see Tullock (1985).

Further, in our model, the bias appears in the following form:

$$bias = \theta - m = \alpha(1 - m).$$

The bias is decreasing in the underlying merit of the patent, implying that a high-merit patent would be less affected by the presumption criterion. An alternative modeling assumption could be considering a constant bias at every level of a patent's merit. However, given the natural bounds of probabilities, the assumption of constant bias is less appealing. Furthermore, we expect the presumption to have a positive influence on the court's belief in favor of patent validity, going beyond the true merit of the patent. In simpler terms, under PV, $\theta - m$ should be positive. Expressing the bias as a specific function of α restricts the generalizability of our analysis. Nonetheless, we have used α in an ad hoc manner: it is necessary to derive a closed-form solution of the litigation contest. However, our analysis of the effect of PV, as discussed in Section 4, does not explicitly consider derivatives with respect to α. Instead, it focuses on how shifting a generic prior θ affects the outcome of the game.

Finally, our framework does not address the possibility of out-of-court settlement. A suitable out-of-court settlement offer can Pareto dominate the outcome of a litigation trial. This observation presents an interesting dilemma: why would litigation trials be chosen over settlements? The literature has uncovered various factors, including, among others, asymmetric information or asymmetric beliefs between disputing parties, and indivisibility of the prize; see Spier (2007) for a comprehensive survey.[23] Our framework does not assume any of these factors, implying there is always potential for a settlement offer to improve outcomes for both sides compared to a trial. In our model, the defendant's only alternative to a trial is accepting a default judgment. We exclude pre-trial settlements to focus on analyzing how presumption bias influences the strategic pursuit of evidence. While we recognize the potential influence of presumption bias on settlement offers, a detailed examination of this aspect falls outside the scope of this paper and is left for future research.

3 The equilibrium analysis

We begin our analysis at the final stage of the game. Consider the subgame in which both players arrive at a trial. The participation cost is now sunk and P and Q simultaneously choose their expenditures (e_P, e_Q) to maximize their respective expected payoffs. As the transfer of damage

[23] See also Bebchuk (1984); Reinganum and Wilde (1986); Spier (1992) for analyses of settlement in litigation. Additionally, for analyses specific to patent litigation, see Meurer (1989); Crampes and Langinier (2002).

compensation takes place only if P wins, the expected payoffs are as follows:

$$\begin{aligned} U_P &= \mathbb{E}_s\left[\pi\left(E_P, E_Q, \theta\right)\right] - e_P - k, \\ U_Q &= -\mathbb{E}_s\left[\pi\left(E_P, E_Q, \theta\right)\right] - e_Q - k, \end{aligned} \tag{5}$$

where $\mathbb{E}_s[\cdot]$ is the expectation operator over the probability distribution of s.

A profile of resources (e_P, e_Q) can lead to four possible events with distinct profiles of evidence (E_P, E_Q), which are (he_p, e_Q), (he_p, he_Q), (e_p, he_Q), and (e_p, e_Q). We denote these events by $event_{h1}$, $event_{hh}$, $event_{1h}$, and $event_{11}$, respectively, such that the subscript ij refers to the event in which the evidence profile is $(ie_P, je_Q), i \in \{h, 1\}, j \in \{h, 1\}$. Because E_P and E_Q are independent, we determine the state-conditional probabilities of these events using the marginal distributions, as described in (3) and (4). Specifically,

$$\Pr\left[event_{h1} \mid s\right] = \begin{cases} \gamma^2 & \text{if } s = V \\ (1-\gamma)^2 & \text{if } s = I \end{cases}, \qquad \Pr\left[event_{hh} \mid s\right] = \begin{cases} \gamma(1-\gamma) & \text{if } s = V \\ \gamma(1-\gamma) & \text{if } s = I \end{cases},$$
$$\Pr\left[event_{1h} \mid s\right] = \begin{cases} (1-\gamma)^2 & \text{if } s = V \\ \gamma^2 & \text{if } s = I \end{cases}, \quad \text{and,} \quad \Pr\left[event_{11} \mid s\right] = \begin{cases} \gamma(1-\gamma) & \text{if } s = V \\ \gamma(1-\gamma) & \text{if } s = I \end{cases}. \tag{6}$$

As already discussed, differently from the beliefs about contest success probabilities that are influenced by court's prior and thus affected by the presumption criterion, the competing parties' beliefs about the state of validity at the time of spending resources are solely guided by the presumption-free prior. We can therefore determine the competing parties' ex-ante belief about $event_{ij}$, denoted by q_{ij}, as

$$q_{ij} := \Pr\left[event_{ij}\right] = \Pr\left[event_{ij} \mid s = V\right] \Pr\left[s = V\right] + \Pr\left[event_{ij} \mid s = I\right] \Pr\left[s = I\right],$$

which gives us

$$\begin{aligned} q_{h1} &= \Pr\left[event_{h1}\right] = m\gamma^2 + (1-m)(1-\gamma)^2, \\ q_{hh} &= \Pr\left[event_{hh}\right] = \gamma(1-\gamma), \\ q_{1h} &= \Pr\left[event_{1h}\right] = m(1-\gamma)^2 + (1-m)\gamma^2, \\ q_{11} &= \Pr\left[event_{11}\right] = \gamma(1-\gamma). \end{aligned}$$

For notational convenience, we define $q_0 := q_{hh} + q_{11} = 2\gamma(1-\gamma)$. Then, P's expected payoff given a profile of resources (e_P, e_Q) is given by

$$
\begin{aligned}
U_P &= \sum_{i\in\{h,1\},j\in\{h,1\}} \pi\left(ie_p, je_Q, \theta\right) \Pr\left[event_{ij}\right] - e_P - k \\
&= \frac{q_{h1}\theta\left(he_P\right)^{\mu}}{(1-\theta)\left(e_Q\right)^{\mu}+\theta\left(he_P\right)^{\mu}} + \frac{q_{1h}\theta\left(e_P\right)^{\mu}}{(1-\theta)\left(he_Q\right)^{\mu}+\theta\left(e_P\right)^{\mu}} + \frac{q_0\theta\left(e_P\right)^{\mu}}{(1-\theta)\left(e_Q\right)^{\mu}+\theta\left(e_P\right)^{\mu}} - e_P - k
\end{aligned}
\tag{7}
$$

To derive the terms in the final expression of (7), we apply that $\pi\left(he_p, he_Q, \theta\right) = \pi\left(e_p, e_Q, \theta\right)$. Similarly, we can write Q's expected payoff as

$$
\begin{aligned}
U_Q &= -\sum_{i\in\{h,1\},j\in\{h,1\}} \pi\left(ie_p, je_Q, \theta\right) \Pr\left[event_{ij}\right] - e_Q - k \\
&= -\frac{q_{h1}\theta\left(he_P\right)^{\mu}}{(1-\theta)\left(e_Q\right)^{\mu}+\theta\left(he_P\right)^{\mu}} - \frac{q_{1h}\theta\left(e_P\right)^{\mu}}{(1-\theta)\left(he_Q\right)^{\mu}+\theta\left(e_P\right)^{\mu}} - \frac{q_0\theta\left(e_P\right)^{\mu}}{(1-\theta)\left(e_Q\right)^{\mu}+\theta\left(e_P\right)^{\mu}} - e_Q - k.
\end{aligned}
\tag{8}
$$

The following lemma characterizes the unique symmetric Nash equilibrium of the subgame. The proof is reported in the Appendix.

Lemma 1. *In the litigation contest, both parties incur the same expenditure, e^c, which is given by*

$$
e^c = \theta\left(1-\theta\right)\mu\Gamma, \tag{9}
$$

where Γ is expressed as a function of m, γ, h, μ, and θ, and is given by

$$
\Gamma\left(m,\gamma,h,\mu,\theta\right) = \frac{q_{h1}h^{\mu}}{\left((1-\theta)+\theta h^{\mu}\right)^2} + q_0 + \frac{q_{1h}h^{\mu}}{\left((1-\theta)h^{\mu}+\theta\right)^2}. \tag{10}
$$

In this probabilistic contest, the legal expenditures are rent-seeking and both parties spend the same amount of resources in equilibrium.[24] Nevertheless, there is a possibility of learning from

[24] This is a well-known result from the large rent-seeking literature building on Tullock (1978; 1997), also when applied to litigation (Parisi, 2002; Parisi et al., 2017; Guerra et al., 2019b; Friehe and Wohlschlegel, 2019; Massenot et al., 2021). The assumption $\mu \leq 1$ ensures that a player's payoff function is globally concave in investment, and the simultaneous first-order conditions yield an interior solution where players' expected payoffs are positive. If μ exceeds one, players may experience negative payoffs at the solution, indicating that refraining from investment is a better strategy. However, zero investment by both players does not constitute an equilibrium; indeed, the game does not always possess a pure-strategy Nash equilibrium in such circumstances (Fu and Wu, 2019, pp. 8). For high values of μ, Baye et al. (1994) prove the existence of a symmetric, fully rent-dissipating, mixed-strategy equilibrium in contests with anonymous CSF when players have finite strategic choices. Alcalde and Dahm (2010) establish that a wide range of contests with anonymous CSF can have an equilibrium akin to that observed in standard all-pay auctions, where rents are fully dissipated. Ewerhart (2015) further characterizes the structural form of these mixed-strategy equilibria. However, we cannot directly apply these findings from the literature since our model features non-anonymous CSF,

evidence because the court's posterior can differ from its prior. The likelihood ratio of patent validity based on evidence, $L^V(E_P,E_Q)$, is $(he^*/e^*)^\mu = h^\mu$ and $(e^*/he^*)^\mu = 1/h^\mu$ in the events $event_{h1}$ and $event_{1h}$, respectively, and remains at 1 in the events $event_{hh}$ and $event_{11}$. We can describe the transition of the court's belief, from its prior to posterior, in the equilibrium path as follows:

$$\pi(E_P,E_Q,\theta) = \begin{cases} \frac{\theta h^\mu}{(1-\theta)+\theta h^\mu} & \text{with probability } q_{h1} \\ \theta & \text{with probability } q_0 \\ \frac{\theta}{(1-\theta)h^\mu+\theta} & \text{with probability } q_{1h} \end{cases}. \tag{11}$$

We denote the ex-ante expected value of the posterior by π^e, which is given by

$$\pi^e = \theta\left[\frac{h^\mu q_{h1}}{(1-\theta)+\theta h^\mu} + q_0 + \frac{q_{1h}}{(1-\theta)h^\mu+\theta}\right] \tag{12}$$

The ex-ante expected payoffs of P and Q at the trial are

$$\mathbb{E}(U_P) = \pi^e - e^c - k,$$
$$\mathbb{E}(U_Q) = -\pi^e - e^c - k.$$

Next, we analyze stage 2. Consider Q's decision to defend. She can either pay the compensation value of 1 or defend a trial, in which case, her expected payoff is $\mathbb{E}(U_Q)$. Therefore, Q defends if $-1 \leq \mathbb{E}(U_Q)$, or, equivalently, if

$$\pi^e \leq 1 - e^c - k. \tag{13}$$

Finally, we analyze stage 1. Consider P's decision to litigate. P receives a zero payoff if she does not litigate. Her payoff from litigation depends on Q's response. In particular, if (13) is not satisfied, then default judgment occurs and P receives a payoff of one. If (13) is satisfied, Q defends a trial, in which case P has a positive expected payoff only if $\mathbb{E}(U_P) \geq 0$, or equivalently, if $\pi^e \geq e^c + k$. Therefore, P litigates in the following two situations: one in which Q submits to a default judgment, which occurs if $\pi^e \geq 1 - e^c - k$; the other in which Q responds by defending a trial, which occurs if $e^c + k < \pi^e < 1 - e^c - k$. The first situation presents a first-mover advantage that P has in this sequential game. Combining the two conditions, we find that P litigates if

$$\pi^e \geq \min\{e^c + k, 1 - e^c - k\}. \tag{14}$$

and we are unaware of any studies exploring the existence and structural form of mixed-strategy equilibrium in contests with high values of μ and non-anonymous CSF. So, we consider $\mu \leq 1$.

We can now fully characterize the equilibrium outcome based on the relationship between the expected posterior π^e and the cost-based thresholds $(e^c + k)$ and $(1 - e^c - k)$. Three different regimes can arise in equilibrium:

- **Litigation trial**: P litigates and Q defends a trial. This arises if $e^c + k \leq \pi^e \leq 1 - e^c - k$.
- **Default judgment**: P litigates and Q pays the damage compensation. This arises if (13) does not hold.
- **No litigation**: P does not litigate. This arises if (14) does not hold.

The following proposition characterizes the equilibrium outcome of the litigation game. The proof follows straightforwardly from the preceding discussion.

Proposition 1. *The equilibrium regimes of the litigation game are characterized as follows:*

(a) If $\pi^e < \min\{e^c + k, 1 - e^c - k\}$, then the no-litigation regime prevails.

(b) If $\pi^e > 1 - e^c - k$, then the default-judgment regime prevails.

(c) If $e^c + k \leq \pi^e \leq 1 - e^c - k$, then the litigation-trial regime prevails. This range can be vacuous if $e^c + k > 1/2$.

The intuition behind Proposition 1 is straightforward. The expected posterior π^e reflects how players perceive P's chance of winning a trial. Therefore, the incentives of P and Q to engage in a trial are increasing and decreasing in π^e, respectively. The litigation-trial regime prevails, if at all, when the expected posterior falls within the two cost-based threshold values $(e^c + k)$ and $(1 - e^c - k)$.

4 Effects of the presumption of validity

The presumption criterion induces a bias to the prior, which is otherwise determined by the underlying merit of a patent. We are interested in investigating how this bias affects the outcome and specific efficiency criteria as we vary the merit of the patent. Measuring these effects can be challenging because of the different roles the bias and the patent's merit play in our framework. The bias affects the court's perception of a patent's validity but not the probability of the state of validity. In contrast, the merit of a patent affects both the court's perception and the probability of the state of validity. Because the true state influences the availability of favorable evidence, the probability of the state of validity moderates the expected return to investing resources for seeking evidence. Therefore, the merit m influences the outcome of the litigation game in two different ways: first, through the prior; and second, by moderating the expected return to investment. The first effect is qualitatively similar to the effect of the bias induced by the presumption of validity—and we are particularly interested to delineate this effect. However, by simply measuring the incremental effect of m, we will also capture the second effect.

To circumvent this challenge, we adopt an indirect approach. We fix the merit m and study how changing a generic prior θ affects various derived parameters of our model. By delineating the effect of a generic prior after controlling for the merit's effect, we get to measure how the presumption-induced bias to the prior would affect the outcome of the game.[25]

To this end, consider first the effect of a generic prior θ on the expected posterior, π^e, and the resources spent, e^c, for a fixed value of m. It follows from (12) that $d\pi^e/d\theta > 0$, implying that the expected posterior π^e is increasing in θ. The difference between the expected posterior π^e and the prior can be expressed as

$$\pi^e(\theta) - \theta = \theta(1-\theta)(h^\mu - 1)\left[\frac{q_{h1}}{(1-\theta)+\theta h^\mu} - \frac{q_{1h}}{(1-\theta)h^\mu + \theta}\right], \tag{15}$$

from which it follows that the expected posterior coincides with the prior at $\theta = 0$, 1, and at some $\widehat{\theta}$ that satisfies $q_{h1}/((1-\theta)+\theta h^\mu) = q_{1h}/((1-\theta)h^\mu + \theta)$. Further, whenever $\widehat{\theta} \in (0,1)$, the posterior is above the prior for $\theta \in (0,\widehat{\theta})$ and below the prior for $\theta \in (\widehat{\theta},1)$.

The resource-spending level e^c changes non-monotonically with respect to θ; $e^c = 0$ at $\theta = 0,1$, and $e^c > 0$ for $\theta \in (0,1)$. In Lemma A.1 in the Appendix, we show that the threshold e^c is increasing in $\theta \in [0, 1/(h^\mu+1)]$ and decreasing in $\theta \in [h^\mu/(h^\mu+1), 1]$. Further, if $h^\mu \leq 2$, then e^c is concave, and consequently, $1 - e^c$ is convex.

In the following analysis, in the interest of tractability, we assume $h^\mu \leq 2$. This assumption implies that e^c has a unique maximum at some prior $\theta \in [1/(h^\mu+1), h^\mu/(h^\mu+1)]$. In Section 5, we discuss how the results would be affected if this assumption is violated.

Assumption 1. $h^\mu \leq 2$.

4.1 The equilibrium regimes

We first examine how shifting the prior between the two presumption scenarios, *PV* and *NP*, affects the equilibrium regimes. Despite the non-monotonic effect of the prior on the resources spent during the trial, we find that the expected trial payoffs of P and Q change monotonically with respect to θ. Consequently, there exist threshold values for the prior delineating players' willingness to engage in a trial. The following lemma documents the observation, with the proof

[25] Note that we can achieve similar findings by studying the derivatives of π^e and e^c with respect to α while keeping the values of m fixed. However, these derivatives involve complex algebraic expressions, hence becoming less tractable. In contrast, our approach enables us to interpret the findings in terms of the shifting effects of the prior.

reported in the Appendix.[26]

Lemma 2. *For any given $k \geq 0$, there exists thresholds $\underline{\theta} \geq 0$ and $\overline{\theta} \geq 0$ such that $\pi^e \geq e^c + k$ if and only if $\theta \geq \underline{\theta}$, and $\pi^e \leq 1 - e^c - k$ if and only if $\theta \leq \overline{\theta}$. If $k = 0$, $\underline{\theta} = 0$ and $\overline{\theta} = 1$. Further, $\underline{\theta}$ increases with k and $\overline{\theta}$ decreases with k.*

It follows from Lemma 2 that the relationship between the expected posterior π^e and the cost-based thresholds $(e^c + k)$ and $(1 - e^c - k)$ can appear in three possible forms, as illustrated in Figure 1. In the first form, occurring if $k = 0$ and displayed in the left panel of Figure 1, $\pi^e \in [e^c, 1 - e^c]$ for all θ, and therefore only the litigation-trial regime prevails. In the second form, displayed in the middle panel of Figure 1, $0 < \underline{\theta} \leq \overline{\theta} < 1$, and there are three different regimes in equilibrium—the no-litigation regime for $\theta \in [0, \underline{\theta})$; the litigation-trial regime for $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$; and the default-judgment regime for $\theta \in \left(\overline{\theta}, 1\right]$. Finally, in the third form, displayed in the right panel of Figure 1, $0 < \overline{\theta} < \underline{\theta} < 1$, and there are two different regimes in equilibrium—the no-litigation regime for $\theta \in \left[0, \overline{\theta}\right)$; and the default-judgment regime for $\theta \in \left(\overline{\theta}, 1\right]$.

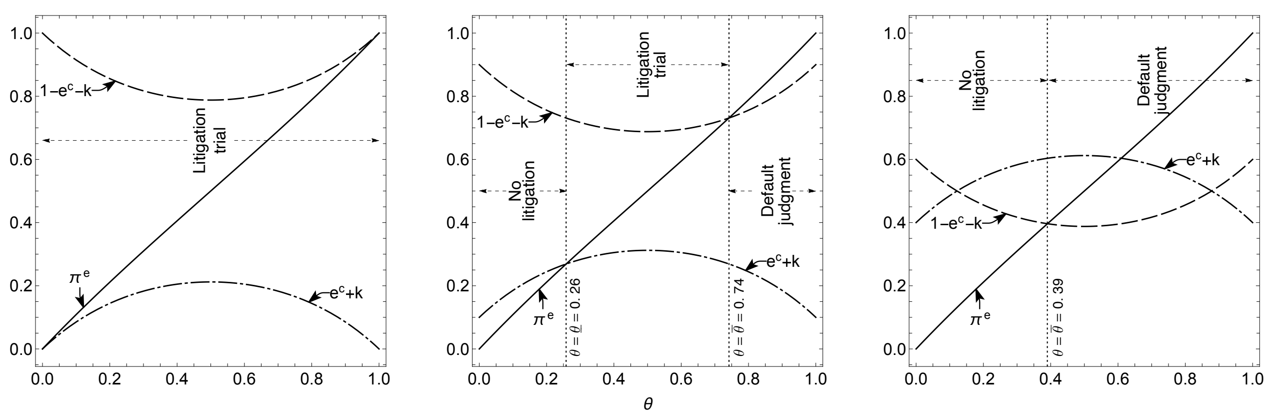

Figure 1: The expected posterior, the cost-based thresholds, and the equilibrium regimes against θ

Notes. All plots in Figure 1 consider the parameter values: $\mu = 0.9$, $h = 2$, $m = 0.5$, $\gamma = 0.75$. Further, we consider $k = 0$, 0.1, and 0.4, in the left-panel, middle-panel, and right-panel diagrams respectively. The continuous curve, the dot-dashed curve, and the dashed curve present π^e, $e^c + k$, and $1 - e^c - k$, respectively.

We can now easily compare the equilibrium regime between the two presumption scenarios—*PV* and *NP*. The presumption of validity moves the prior θ upward, i.e., from m to $\alpha + (1 - \alpha) m$, which takes a value in $(m, 1)$. As the no-litigation regime prevails for $\theta \in \left[0, \min\left\{\underline{\theta}, \overline{\theta}\right\}\right)$ and the default-judgment regime prevails for $\theta \in \left(\overline{\theta}, 1\right]$, it follows that an increase in θ will reduce the possibility of the no-litigation regime and increase the possibility of the default-judgment regime. The effect on the possibility of the litigation-trial regime is ambiguous: it depends on the relative extent of the bias in comparison to the thresholds $\underline{\theta}$ and $\overline{\theta}$.

[26] We have not used Assumption 1 in proving this lemma.

The following proposition summarizes the above observations. The proof trivially follows from the preceding discussion.

Proposition 2. *Consider, as a point of comparison, NP as the default scenario. The introduction of the presumption of validity expands the set of parameter values for which the default-judgment regime prevails in equilibrium and shrinks the set of parameter values for which the no-litigation regime prevails in equilibrium. Further, the set of parameter values for which the litigation-trial regime prevails in equilibrium can either expand or shrink.*

Figure 2 plots the equilibrium regimes in the (θ,μ) space. To understand the findings of Proposition 2, consider a patent of merit $m = 0.4$ and $\alpha = 0.4$. If there is no presumption, the prior is $\theta = m = 0.4$, which is represented by the dotted line in Figure 2. At $\theta = 0.4$, the litigation-trial regime exists for low values of μ. With the presumption of validity, the prior is $\theta = 0.4 + 0.6 \times 0.4 = 0.64$, which is represented by the dashed line in the figure. At $\theta = 0.64$, the default-judgment regime prevails over a larger range of values of μ and the no-litigation regime ceases to occur.

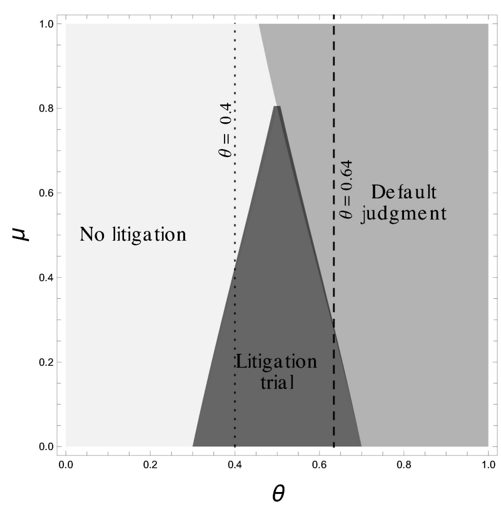

Figure 2: The equilibrium regimes in the (θ,μ) space

Notes. Figure 2 considers the parameter values: $h = 2$, $\gamma = 0.9$, $m = 0.4$ and $k = 0.3$. The equilibrium regimes are plotted in the (θ,μ) space. The dotted and the dashed line represent the priors $\theta = 0.4$ and $\theta = 0.64$, respectively.

Proposition 2 shows that the presumption of validity affects the existence of the litigation-trial regime in equilibrium in an ambiguous way. To explore further, we next focus on two interesting features of the litigation-trial regime. First, a litigation trial is socially costly, mainly because resources are dissipated for non-productive activities. Second, there is a scope for learning through evidence produced in the trial. The learning moderates the possibility of making errors of judgment, i.e., rejecting the validity of a valid patent and accepting the validity of an invalid patent during a trial. In the following two subsections, we analyze the effects of the presumption of validity on these two features of the litigation-trial regime, namely resource dissipation and possible errors of judgment.

4.2 Resource dissipation

The total resource dissipation is given by

$$R = 2e^c + 2k = 2\theta(1-\theta)\mu\Gamma + 2k.$$

The presumption criterion affects R only through the prior θ. We first study how a generic prior θ affects R for a fixed value of m. It follows from Lemma A.1 that R is increasing in $\theta \le 1/(h^\mu+1)$ and decreasing in $\theta \ge h^\mu/(h^\mu+1)$. Further, under Assumption 1, R has its maximum at a unique θ, which we denote by θ^R. Specifically,

$$\theta^R := \underset{\theta\in[0,1]}{\arg\max} R. \tag{16}$$

Under Assumption 1, it follows from the first-order condition of (16) that θ^R uniquely solves the following:

$$\left[\frac{q_{h1}h^\mu(h^\mu+1)}{((1-\theta)+\theta h^\mu)^3}\left(\frac{1}{h^\mu+1}-\theta\right)+2q_0\left(\frac{1}{2}-\theta\right)+\frac{q_{1h}h^\mu(h^\mu+1)}{((1-\theta)h^\mu+\theta)^3}\left(\frac{h^\mu}{h^\mu+1}-\theta\right)\right]=0. \tag{17}$$

The following lemma documents the relationship between a patent's merit m and θ^R.

Lemma 3. *Fix m and consider R as a function of a generic prior θ. Under Assumption 1, R is uniquely maximized at some $\theta^R \in (0,1)$. Further, $\theta^R < 1/2$ if $m > 1/2$, and $\theta^R > 1/2$ if $m < 1/2$. And, $\theta^R = m$ if $m = 1/2$.*

Figure 3 plots θ^R against m for a certain parametric specification. Suppose that the true state does not influence evidence production, i.e., $h = 1$. Then, it follows from (10) that $\Gamma = 1$ and $R = 2\theta(1-\theta)\mu + 2k$, which is maximized at $\theta = 1/2$. Therefore, for every m, θ^R remains at $1/2$; in other words, if the true state did not influence the availability of favorable evidence, both parties expend their resources to the full extent when the court deems both states equally likely.

To understand the findings of Lemma 3, consider what might happen if $h > 1$. We first argue for the case $m > 1/2$. As $h > 1$, P finds it easier to produce favorable evidence compared to Q. The asymmetry discourages Q to expend resources, and, consequently, both parties' contest effort levels decrease. This discouragement effect can be partially mitigated if the court perceived the state of patent validity to be less likely, which would give Q an incentive to expend resources. This is the reason why θ^R, the resource-dissipation-maximizing prior value, must be less than $1/2$ whenever $m > 1/2$. Similarly, for $m < 1/2$, the asymmetric cost of evidence production discourages P to expend resources, and this discouragement effect can only be partially compensated if the court's prior is favorably biased toward the state of patent validity, resulting in $\theta^R > 1/2$.

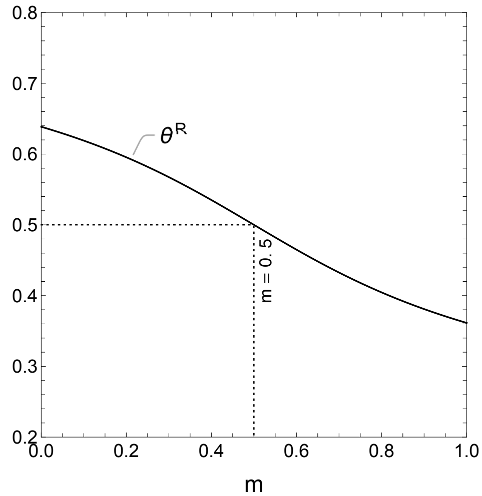

Figure 3: θ^R against m

Notes. Figure 3 considers the following parameter values: $\mu = 1$, $h = 2$, $\gamma = 0.9$.

Note that the presumption of validity not only changes the value of R, but also the type of the equilibrium regime. If, for some parametric specification, there is litigation-trial regime in equilibrium in one presumption scenario but not the other, then comparing resource dissipation between the two scenarios is straightforward. In contrast, if there is litigation trial in equilibrium under both PV and NP, then measuring the effect of the presumption on R is more complex.

To look into the effect of the presumption of validity, we now focus on the parameter space for which there will be litigation trial under both PV and NP and compare R between the two scenarios as m changes. Assume, without loss of generality, NP to be the default scenario and consider patents with $m > 1/2$. Then, $\theta^R < 1/2 < m$. The presumption of validity moves the value of the prior upward in the range $(m, 1)$. As R is decreasing in $\theta \in [\theta^R, 1]$, the presumption of validity will only reduce R. For $m < 1/2$, $m < 1/2 < \theta^R$. The effect on R is ambiguous; it can increase or decrease, depending on values of α and m, which determine to what extent the presumption-driven prior θ increases from m. Using the fact that R is concave under Assumption 1, the following proposition shows that the aggregate resource dissipation R is strictly lower under NP than under PV if and only if m is below a threshold lower than $1/2$.

Proposition 3. *Consider the range of parameter values for which the litigation-trial regime prevails in equilibrium in both the presumption scenarios, PV and NP. Further, consider, as a point of comparison, NP as the default scenario. There exists a threshold* $m_{PV}^{R} \in (0, 1/2)$ *such that the introduction of PV will increase (decrease) the aggregate resource dissipation R if m is less (greater) than* m_{PV}^{R}.

The intuition behind Proposition 3 is straightforward. As m deviates from 1/2 in either direction, one of the two parties finds it easier to produce evidence, creating an imbalance that generally discourages both parties from investing. For high-merit patents, the presumption of validity exacerbates the existing imbalance. In contrast, for low-merit patents, the presumption partially mitigates the imbalance. Therefore, the threshold merit level at which the presumption can effectively neutralize the discouraging effects caused by asymmetric merit is always below 1/2.

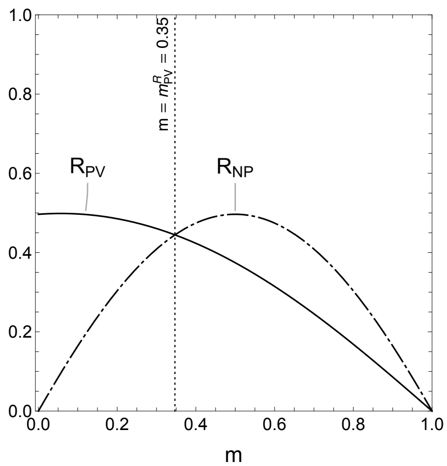

Figure 4: R against m

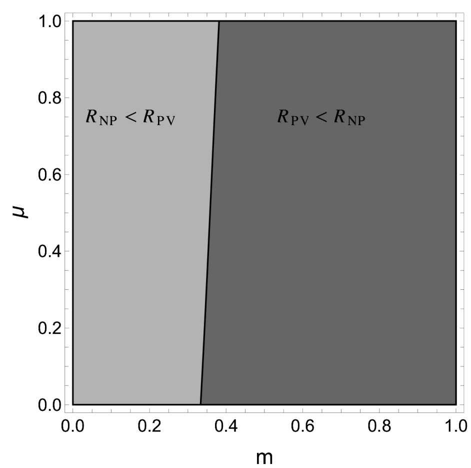

Figure 5: Comparison of R between PV and NP in the (m, μ) space

Notes. Figure 4 considers the parameter values: $k = 0$, $\mu = 1$, $h = 2$, $\gamma = 0.9$, and $\alpha = 0.5$. R_{PV} (the continuous curve) and R_{NP} (the dot-dashed curve) denote the resource dissipation R under $\theta = \theta_{PV}$ and $\theta = m$, respectively and we have $R_{NP} < R_{PV}$ if $m < m_{PV}^{R} = 0.35$. Figure 5 considers the same parameter values (relaxing μ) and compares R_{PV} with R_{NP} in the (m, μ) space.

In Figure 4, we plot R against m under PV and NP. Figure 5 compares R between PV and NP in the (m, μ) space. In Figures 4 and 5, we consider $k = 0$ so that the litigation-trial regime prevails in equilibrium for all possible priors under both PV and NP.

4.3 Error of judgment

An error of judgment in the court's decision-making occurs if the outcome of the litigation game results in either rejecting a patent's validity when it is valid (equivalent to a false negative or type-II error in statistical binary classification), or accepting a patent's validity when it is invalid (equivalent to a false positive or type-I error). The aggregate probability of making an error of judgment in litigation is given by[27]

$$\begin{aligned}
E &= \Pr[P \text{ wins } \cap s = I] + \Pr[Q \text{ wins } \cap s = V] \\
&= \Pr[P \text{ wins } \mid s = I]\Pr[s = I] + \Pr[Q \text{ wins } \mid s = V]\Pr[s = V] \qquad (18) \\
&= (1-m)\left[\sum_{i\in\{h,1\},j\in\{h,1\}} \pi(ie_p, je_Q, \theta)\Pr\left[event_{ij} \mid s = I\right]\right] + \\
&\quad + m\left[\sum_{i\in\{h,1\},j\in\{h,1\}} (1-\pi(ie_p, je_Q, \theta))\Pr\left[event_{ij} \mid s = V\right]\right] \\
&= m + \sum_{i\in\{h,1\},j\in\{h,1\}} \pi(ie_p, je_Q, \theta)\left[(1-m)\Pr\left[event_{ij} \mid s = I\right] - m\Pr\left[event_{ij} \mid s = V\right]\right] \qquad (19)
\end{aligned}$$

Using (2) and the fact that $e_P = e_Q = e^c$ in equilibrium, we can replace $\pi(ie_p, je_Q, \theta)$ by $\theta i^{\mu}/[(1-\theta)j^{\mu} + \theta i^{\mu}]$. Further, in $event_{hh}$ and $event_{11}$, we have $i = j$, and it follows that $\pi(ie_p, je_Q, \theta) = \theta$. Using the state-conditional probabilities of various events from (6), (19) further reduces to the following expression:

$$\begin{aligned}
E = m &+ \frac{\theta h^{\mu}\left[(1-m)(1-\gamma)^2 - m\gamma^2\right]}{(1-\theta)+\theta h^{\mu}} \\
&+ \frac{\theta\left[(1-m)\gamma^2 - m(1-\gamma)^2\right]}{(1-\theta)h^{\mu}+\theta} + 2\theta\left[(1-2m)\gamma(1-\gamma)\right]. \qquad (20)
\end{aligned}$$

As shown in (18), E is a weighted sum of two conditional probabilities, measuring chances of a false positive ($\Pr[P \text{ wins } \mid s = I]$) and false negative ($\Pr[Q \text{ wins } \mid s = V]$). Further, the weights are not constant across merits. The weight associated with a false positive is high for low-merit patents and low for high-merit patents.[28] The following lemma documents some useful properties of E.

Lemma 4. *Fix m and consider E as a function of a generic prior θ. Then,*

[27] Here and in what follows, we slightly ease the notation by denoting the measure of judgment error as E without any subscript. Previously, we used E_i with a subscript i to indicate the evidence produced by party i.

[28] Previous studies have often assigned exogenous weights to these two types of errors. In civil offenses, the two errors are commonly treated equally, whereas in criminal offenses, a false positive case is considered as more serious (Burtis et al., 2017; Clermont and Sherwin, 2002). However, by focusing on the probability of committing any error, we effectively make these weights endogenous. For example, a false positive case can only occur when the state is invalid. Consequently, the probability of encountering a false positive case is naturally higher for low-merit patents, which are more likely to be invalid. Additionally, assigning pre-fixed, unequal weights to the two errors across patents of all merits would introduce an additional source of bias in favor of one party, making it more challenging to disentangle the effect of the biased prior.

(i) There exists thresholds $\underline{m} < 1/2 < \overline{m}$ such that for any $m \leq \underline{m}$, E is increasing in $\theta \in [0,1]$, and for any $m \geq \overline{m}$, E is decreasing in $\theta \in [0,1]$.

(ii) Further, for $m \leq 1/2$, E is increasing in θ for all $\theta \in [1/2,1]$ and for $m \geq 1/2$, E is decreasing in θ for all $\theta \in [0,1/2]$. It follows that for $m = 1/2$, E reaches its minimum at $\theta = 1/2$.

Part (i) of the above lemma illustrates the effect of the presumption criterion in clear terms when m takes sufficiently high or low values. For example, for $m \geq \overline{m}$, the presumption of validity only reduces an error of judgment by shifting the prior upward. The opposite effect will be realized for $m \leq \underline{m}$. For intermediate values of m, the analysis becomes complex.

A change in θ, after controlling for merit m, affects E in two distinct ways. The first is a *direct* effect. An increase in the prior θ leads to a qualitatively similar increase in the posterior π, which raises the probability of false positive and reduces the probability of a false negative. The second is an *indirect* effect. When θ gets close to $1/2$, because of the high intensity of competition, both parties spend a high volume of resources in unearthing new evidence. The probabilities of both a false positive and a false negative decrease with high evidence-seeking incentives.

To understand the direction of the combined effect, consider first the case of a low-merit patent, with m below $1/2$. Because a low-merit patent has higher weight on the false positive than on the false negative, the direct effect on E described above is increasing in θ. In contrast, the indirect effect due to evidence-seeking incentive does not follow a monotone path. As θ approaches $1/2$, the indirect effect dampens E, but then raises it as θ moves further away from $1/2$. Together, for sufficiently low values of m, the direct effect dominates the indirect because of the high weight on the conditional probability of false positive, and E increases with θ over its full range. For m close to $1/2$, the indirect effect can dominate the direct effect and E might be decreasing in $\theta \in (m, 1/2)$. However, for $\theta > 1/2$, both effects move in the same direction, which explains the second part of the above lemma. In the case of a high-merit patent with $m > 1/2$, the indirect effect works the same way as above but the direct effect works in the opposite direction. This is because a high-merit patent puts higher weight on the false negative than on the false positive, and an increase in θ reduces the probability of a false negative and raises the probability of a false positive.

Next, to study the effect of the presumption of validity, we vary m and compare E between the two scenarios, *PV* and *NP*. Because a low-merit patent ($m < 1/2$) puts more weight on the probability of a false positive than a high-merit patent ($m > 1/2$) does, the direct effect of the presumption bias typically increases the error of judgment by a higher margin for a low-merit patent. However, when m is below but close to $1/2$, the evidence-seeking incentive is high even without the presumption bias, and the marginal effect of increasing the competition intensity by biasing the prior is relatively low. Thus, the positive indirect effect of the bias by increasing the evidence-seeking incentive is always dominated by the negative direct effect of increasing the probability of a false positive for all patents with $m < 1/2$. The following proposition shows that for all m below a threshold that is weakly higher than $1/2$, PV is always associated with a higher error of judgment compared to NP.

Proposition 4. *Consider the range of parameter values for which the litigation-trial regime prevails in equilibrium in both the presumption scenarios, PV and NP. Further, consider, as a point of comparison, NP as the default scenario. There exists a threshold* $m^E_{PV} \in (1/2, 1)$ *such that the introduction of PV will increase the error of judgment E if m is less than* m^E_{PV}.

It is worth noting that unlike Proposition 3, Proposition 4 does not provide a necessary and sufficient threshold-based result for comparing E between the two regimes. This is because the two effects of the presumption bias described earlier, the direct effect of changing probabilities of a false positive and false negative and the indirect effect of changing evidence-seeking incentive, can move at different rates causing E to change in a non-monotone way.

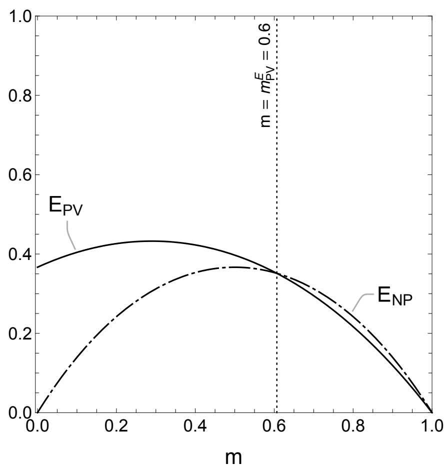

Figure 6: E against m

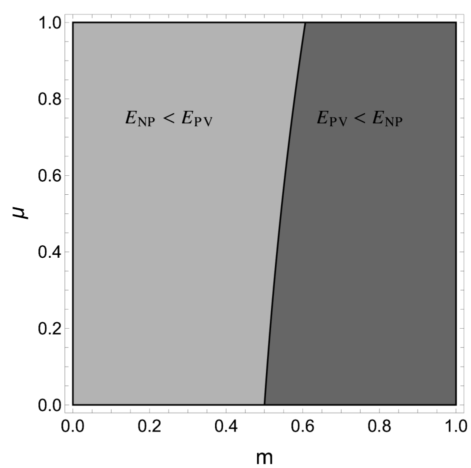

Figure 7: Comparison of E between PV and NP in the (m, μ) space

Notes. Figure 6 considers the parameter values: $k = 0$, $\mu = 1$, $h = 2$, $\gamma = 0.9$, and $\alpha = 0.5$. E_{PV} (the continuous curve) and E_{NP} (the dot-dashed curve) denote the error value E, computed at the priors $\theta = \theta_{PV}$ and $\theta = m$, respectively and we have $E_{NP} < E_{PV}$ if $m < m^E_{PV} = 0.6$. Figure 7 considers the same parameter values (relaxing μ), and compares E_{PV} with E_{NP} in the (m, μ) space.

Figures 6 and 7 illustrate the proposition's findings using a numerical example. In Figure 6, we plot E against m under PV and NP. Figure 7 compares E between PV and NP in the (m, μ) space. We consider $k = 0$ so that the litigation-trial regime prevails in equilibrium for all possible priors under both PV and NP.

5 Discussion

In this section, we briefly discuss the efficiency concerns associated with the presumption criterion and the role of Assumption 1 in our analysis. We conclude with suggestions for future research.

A litigation trial is a non-productive contest. It only results in a transfer of ownership rights. The notion of economic efficiency is built around the concept of allocative efficiency, which deals with how costly resources can be optimally allocated to productive activities in order to achieve maximum benefits. By this notion, spending resources in any non-productive contest is an inefficient activity (Tullock, 1967; Krueger, 1974; Bhagwati, 1982; Guerra et al., 2019a). Our framework, however, explores a different effect of investing resources in a litigation contest. It helps unearthing new evidence that contributes to better judgment and proper allocation of ownership rights. An error of judgment is costly to society, just as resource dissipation is (Buzzacchi and Scellato, 2008). An evaluation of the presumption criterion should therefore include both dimensions, resource dissipation and error of judgment.

In this context, the findings of Propositions 3 and 4 reflect the potential trade-off associated with the presumption criterion when there is sufficient uncertainty about the patent's merit. In particular, for patents with merit m between the two thresholds, m^R_{PV} and m^E_{PV}, placing on either side of $1/2$, the presumption has contrasting effects. In this case, introducing the presumption will likely raise the error of judgment, but decrease the resource dissipation. This is because, for m close to $1/2$, the intensity of competition is at its peak. The bias induced by the presumption likely reduces the intensity, because of which fewer resources would be dissipated. However, it would also reduce the incentive to gather new evidence, which would rather be highly needed especially when there is great uncertainty about the patent's merit.

This finding suggests that we should pay careful attention to the application of the presumption, especially in contexts where the examination of patent applications is complex and invalid patents are granted more frequently by the patent office (e.g., high technology sectors). Examining patent applications is becoming increasingly difficult for several reasons, including the growing number of applications and the budgetary constraints the patent office faces. Granting of invalid patents is an ever-growing reality and the possibility of resolving disputes through legal proceedings is an inevitable consequence (Buzzacchi and Scellato, 2008; Farrell and Shapiro, 2008; de Rassenfosse et al., 2021). Accordingly, in these contexts where m is close to $1/2$, our findings support the arguments against the application of the presumption, which should not be accepted, or even dismissed *tout court*.

Let us now discuss the role of Assumption 1 in our analysis. It ensures concavity of the resource-spending level, which allows us to study comparative static effects on resource dissipation and judgment error in a tractable form.[29] The assumption is not necessary for characterizing the equilibrium regimes or for determining the effect of shifting the prior on the equilibrium regime. Relaxing this assumption would impact Lemma 3, the proof of which relies on Assumption 1 to ensure the uniqueness of θ^R. If the assumption is violated, e^c is still increasing for $\theta \leq 1/(1+h^{\mu}(\mu-1))$ and is decreasing for $\theta \geq h^{\mu}/(h^{\mu}+\mu-1)$; however, there could be multiple local maxima for the intermediate values of θ. Therefore, in the absence of Assumption 1, we will still observe that the presumption is going to increase (decrease) the aggregate resource dissipation for sufficiently low-merit (high-merit) patents; however, the uniqueness of the threshold documented in Proposition 3 will not hold.

Our analysis provides novel insights into how the presumption of validity affects the frequency of litigation trials, and reveals its countervailing efficiency effects when there is uncertainty about the patent's merit. Our findings come from analyzing partial effects, under the maintained hypothesis that the patent has been produced and granted. Some significant factors are absent from our analysis, which can be extended in various directions. Future research can analyze whether changes in presumption criteria affect the level of R&D investments and the frequency of patent applications, and what implications can be derived for growth and innovation. Also, it could be interesting to study the effects of litigation outcomes on the market structure, under different presumption criteria. In some situations, a favorable outcome to the plaintiff or the defendant can increase or decrease the market competition, with additional gain or loss in consumer surplus. Finally, our setup can be extended to analyze the interaction of legal presumptions with other institutional variables, including the standard of proof and the structure of the patent-granting process.

[29] Note that the assumption is only a sufficient condition to achieve concavity of e^c. While constructing numerically examples, we find that h^{μ} must be sufficiently high (around 40) to violate concavity. Nevertheless, the expected payoffs of P and Q change monotonically even when concavity of e^c is violated.

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Appendix

The appendix contains the proofs that are omitted in the main text. We will begin with documentation of three additional results, Lemma A.1, Lemma A.2, and Lemma A.3, that will be useful in proving our main findings. Lemma A.1 documents how e^c changes with respect to θ, and Lemma A.2 and Lemma A.3 document how θ affects the difference between the expected posterior π^e and the two cost thresholds (e^c+k) and $(1-e^c-k)$, respectively.

Lemma A.1. *e^c is increasing in $\theta \in [0, 1/(h^\mu+1)]$ and decreasing in $\theta \in [h^\mu/(h^\mu+1), 1]$. Further, if $h^\mu \leq 2$, then e^c is concave in $\theta \in [0,1]$.*

Proof of Lemma A.1. Observe that

$$\frac{de^c}{d\theta} = \mu \left[q_{h1} h^\mu \frac{d}{d\theta} \frac{\theta(1-\theta)}{((1-\theta)+\theta h^\mu)^2} + q_0 \frac{d\theta(1-\theta)}{d\theta} + q_{1h} h^\mu \frac{d}{d\theta} \frac{\theta(1-\theta)}{((1-\theta)h^\mu+\theta)^2} \right],$$

which, after simplifying, reduces to

$$\frac{de^c}{d\theta} = \mu \left[\frac{q_{h1} h^\mu (h^\mu+1)}{((1-\theta)+\theta h^\mu)^3} \left(\frac{1}{h^\mu+1} - \theta \right) + 2q_0 \left(\frac{1}{2} - \theta \right) + \frac{q_{1h} h^\mu (h^\mu+1)}{((1-\theta)h^\mu+\theta)^3} \left(\frac{h^\mu}{h^\mu+1} - \theta \right) \right].$$

Because $0 < 1/(h^\mu+1) < 1/2 < h^\mu/(h^\mu+1) < 1$, $(de^c/d\theta)$ is strictly positive for all $\theta \leq 1/(h^\mu+1)$ and $(de^c/d\theta)$ is strictly negative for all $\theta \geq h^\mu/(h^\mu+1)$, these observations together prove the first part of the lemma. Further, it follows that the global maximum lies in $[1/(h^\mu+1), h^\mu/(h^\mu+1)]$ and multiple local optima might exist for $\theta \in [1/(h^\mu+1), h^\mu/(h^\mu+1)]$, depending on the curvature of e^c. To study the curvature, we examine the second-order derivative.

$$\frac{d^2e^c}{d\theta^2} = \mu \left[q_{h1} h^\mu \frac{d}{d\theta} \frac{1-\theta(h^\mu+1)}{((1-\theta)+\theta h^\mu)^3} + q_0 \frac{d(1-2\theta)}{d\theta} + q_{1h} h^\mu \frac{d}{d\theta} \frac{h^\mu - \theta(h^\mu+1)}{((1-\theta)h^\mu+\theta)^3} \right],$$

which, after simplifying, reduces to

$$\frac{d^2e^c}{d\theta^2} = \mu \left[\frac{2q_{h1} h^\mu (h^{2\mu}-1)}{((1-\theta)+\theta h^\mu)^4} \left(\theta - \frac{2h^\mu-1}{h^{2\mu}-1} \right) - 2q_0 + \frac{2q_{1h} h^\mu (h^{2\mu}-1)}{((1-\theta)h^\mu+\theta)^4} \left(\frac{h^\mu(h^\mu-2)}{h^{2\mu}-1} - \theta \right) \right].$$

Observe that if $h^\mu \leq 2$, then $(2h^\mu-1)/(h^{2\mu}-1) \geq 1$ and $h^\mu(h^\mu-2)/(h^{2\mu}-1) \leq 0$, which together imply $(d^2e^c/d\theta^2)$ is negative, or equivalently, e^c is globally concave for $\theta \in [0,1]$. □

Lemma A.2. *Define $F(\theta) := \pi^e - (1-e^c-k)$. $F(\theta)$ is strictly increasing in $\theta \in [0,1]$.*

Proof of Lemma A.2. Observe that

$$\begin{aligned}
F(\theta)-k &= \pi^e-(1-e^c)=e^c-(1-\pi^e)\\
&= q_{h1}\left(\frac{\theta(1-\theta)\mu h^\mu}{((1-\theta)+\theta h^\mu)^2}-\frac{(1-\theta)}{(1-\theta)+\theta h^\mu}\right)\\
&+q_0(\theta(1-\theta)\mu-(1-\theta))+q_{1h}\left(\frac{\theta(1-\theta)\mu h^\mu}{((1-\theta)h^\mu+\theta)^2}-\frac{(1-\theta)h^\mu}{(1-\theta)h^\mu+\theta}\right)\\
&=-\frac{q_{h1}(1-\theta)((1-\theta)+h^\mu\theta(1-\mu))}{((1-\theta)+\theta h^\mu)^2}-q_0(1-\theta)(1-\theta\mu)\\
&-\frac{q_{1h}h^\mu(1-\theta)(h^\mu(1-\theta)+\theta(1-\mu))}{((1-\theta)h^\mu+\theta)^2}.
\end{aligned}$$

The first-order derivatives of the three components of are as follows:

$$\begin{aligned}
&(i)\frac{d}{d\theta}\left(-\frac{q_{h1}(1-\theta)((1-\theta)+h^\mu\theta(1-\mu))}{((1-\theta)+\theta h^\mu)^2}\right)\\
&=\frac{q_{h1}}{((1-\theta)+\theta h^\mu)^3}\left[(2h^\mu(1-\theta)\mu)+h^\mu(1-\mu)((1-\theta)+\theta h^\mu)\right];\\
&(ii)\frac{d}{d\theta}(-q_0(1-\theta)(1-\theta\mu))=q_0((1-\theta\mu)+\mu(1-\theta));\\
&(iii)\frac{d}{d\theta}\left(-\frac{q_{1h}h^\mu(1-\theta)(h^\mu(1-\theta)+\theta(1-\mu))}{((1-\theta)h^\mu+\theta)^2}\right)\\
&=\frac{q_{1h}}{((1-\theta)h^\mu+\theta)^2}\left[2(h^\mu-1)\theta(1-\theta)\mu+((1-\theta\mu)+\mu(1-\theta))((1-\theta)h^\mu+\theta)\right];
\end{aligned}$$

each of these derivatives are strictly positive when $0<\mu\leq 1$. Therefore, $F(\theta)+k$, and equivalently, $F(\theta)$ is strictly increasing in $\theta\in[0,1]$. □

Lemma A.3. *Define* $G(\theta):=\pi^e-e^c-k$. $G(\theta)$ *is strictly increasing in* $\theta\in[0,1]$.

Proof of Lemma A.3. Observe that

$$\begin{aligned}
G(\theta)+k =& q_{h1}\left(\frac{\theta h^\mu}{(1-\theta)+\theta h^\mu}-\frac{\theta(1-\theta)\mu h^\mu}{((1-\theta)+\theta h^\mu)^2}\right)\\
&+q_0(\theta-\theta(1-\theta)\mu)+q_{1h}\left(\frac{\theta}{(1-\theta)h^\mu+\theta}-\frac{\theta(1-\theta)\mu h^\mu}{((1-\theta)h^\mu+\theta)^2}\right)\\
=&\frac{q_{h1}h^\mu\theta(h^\mu\theta+(1-\mu)(1-\theta))}{((1-\theta)+\theta h^\mu)^2}+q_0(\theta-\theta(1-\theta)\mu)+\frac{q_{1h}\theta(\theta+h^\mu(1-\mu)(1-\theta))}{((1-\theta)h^\mu+\theta)^2}.
\end{aligned}$$

The first-order derivatives of the three components of are as follows:

$$
\begin{aligned}
(i)\frac{d}{d\theta}&\frac{q_{h1}h^{\mu}\theta\left(h^{\mu}\theta+(1-\mu)(1-\theta)\right)}{\left((1-\theta)+\theta h^{\mu}\right)^{2}}\\
=&\frac{q_{h1}h^{\mu}}{\left((1-\theta)+\theta h^{\mu}\right)^{3}}\left[\left(2\theta\mu\left(h^{\mu}-1\right)(1-\theta)\right)+\left(2\theta\mu+(1-\mu)\right)\left((1-\theta)+\theta h^{\mu}\right)\right];\\
(ii)\frac{d}{d\theta}&q_{0}\left(\theta-\theta(1-\theta)\mu\right)=q_{0}\left(2\theta\mu+(1-\mu)\right);\\
(iii)\frac{d}{d\theta}&\frac{q_{1h}\theta\left(\theta+h^{\mu}(1-\mu)(1-\theta)\right)}{\left((1-\theta)h^{\mu}+\theta\right)^{2}}\\
=&\frac{q_{1h}}{\left((1-\theta)h^{\mu}+\theta\right)^{3}}\left[2\theta\mu+h^{\mu}(1-\mu)\left((1-\theta)h^{\mu}+\theta\right)\right];
\end{aligned}
$$

each of these derivatives are strictly positive when $0<\mu\leq 1$. Therefore, $G(\theta)+k$, and equivalently, $G(\theta)$ is strictly increasing in $\theta\in[0,1]$. □

Proof of Lemma 1. At the trial stage, the participation cost is sunk, and therefore it does not affect the optimal choice of resource spending. The first-order condition of maximizing U_P with respect to e_P gives

$$\frac{q_{h1}\theta(1-\theta)\mu h\left(he_{P}\right)^{\mu-1}\left(e_{Q}\right)^{\mu}}{\left((1-\theta)\left(e_{Q}\right)^{\mu}+\theta\left(he_{P}\right)^{\mu}\right)^{2}}+\frac{q_{1h}\theta(1-\theta)\mu\left(e_{P}\right)^{\mu-1}\left(he_{Q}\right)^{\mu}}{\left((1-\theta)\left(he_{Q}\right)^{\mu}+\theta\left(e_{P}\right)^{\mu}\right)^{2}}+\frac{q_{0}\theta(1-\theta)\mu\left(e_{P}\right)^{\mu-1}\left(e_{Q}\right)^{\mu}}{\left((1-\theta)\left(e_{Q}\right)^{\mu}+\theta\left(e_{P}\right)^{\mu}\right)^{2}}=1. \tag{A.1}$$

Similarly, the first-order condition of maximizing U_Q with respect to e_Q gives

$$\frac{q_{h1}\theta(1-\theta)\mu\left(he_{P}\right)^{\mu}\left(e_{Q}\right)^{\mu-1}}{\left((1-\theta)\left(e_{Q}\right)^{\mu}+\theta\left(he_{P}\right)^{\mu}\right)^{2}}+\frac{q_{1h}\theta(1-\theta)\mu h\left(e_{P}\right)^{\mu}\left(he_{Q}\right)^{\mu-1}}{\left((1-\theta)\left(he_{Q}\right)^{\mu}+\theta\left(e_{P}\right)^{\mu}\right)^{2}}+\frac{q_{0}\theta(1-\theta)\mu\left(e_{P}\right)^{\mu}\left(e_{Q}\right)^{\mu-1}}{\left((1-\theta)\left(e_{Q}\right)^{\mu}+\theta\left(e_{P}\right)^{\mu}\right)^{2}}=1. \tag{A.2}$$

There is a symmetric solution $e_P=e_Q=e^c$, satisfying the two first-order conditions simultaneously, such that

$$\frac{q_{h1}\theta(1-\theta)\mu h^{\mu}}{\left((1-\theta)+\theta h^{\mu}\right)^{2}}+\frac{q_{1h}\theta(1-\theta)\mu h^{\mu}}{\left((1-\theta)h^{\mu}+\theta\right)^{2}}+q_{0}\theta(1-\theta)\mu=e^{c}. \tag{A.3}$$

After simplifying, (A.3) reduces to $e^c=\theta(1-\theta)\mu\mathit{\Gamma}(m,\gamma,h,\mu,\theta)$, where $\mathit{\Gamma}(m,\gamma,h,\mu,\theta)$ is given by (10). It can be shown that the solution of the first-order condition also satisfies the second-order condition when $\mu\leq 1$. To see this, note that P's expected payoff, U_P, is a linear combination of terms of the form $xe_P^{\mu}/\left(ye_Q^{\mu}+xe_P^{\mu}\right)$ for different expressions of $x,y>0$, with positive coefficients, and $(-e_P)$. The second-order derivative of U_P will, therefore, be a linear combination of terms of the following forms with positive coefficients:

$$\frac{d^2}{de_P^2}\frac{xe_P^{\mu}}{\left(ye_Q^{\mu}+xe_P^{\mu}\right)}=\frac{xy\mu e_P^{2\mu-2}e_Q^{\mu}\left[y(\mu-1)-x(\mu+1)\right]}{\left(ye_Q^{\mu}+xe_P\right)^3},$$

which is strictly negative for any $x,y>0$ if $e_p>0$, $e_Q>0$, and $\mu\leq 1$. Therefore, U_P is globally concave for $e_p>0$, $e_Q>0$.

Similarly, Q's expected payoff, U_Q, is a linear combination of terms of the form $xe_P^{\mu}/\left(ye_Q^{\mu}+xe_P^{\mu}\right)$ for different expressions of $x,y>0$, with negative coefficients, and $(-e_Q)$. Therefore, the second-order derivative of U_Q will be a linear combination of terms of the following forms with negative coefficients:

$$\frac{d^2}{de_Q^2}\frac{xe_P^{\mu}}{\left(ye_Q^{\mu}+xe_P^{\mu}\right)}=\frac{xy\mu e_P^{\mu}e_Q^{2\mu-2}\left[y(\mu+1)+x(1-\mu)\right]}{\left(ye_Q^{\mu}+xe_P\right)^3},$$

which is strictly positive for any $x,y>0$ if $e_p>0$, $e_Q>0$, and $\mu\leq 1$. Because of the negative coefficients, the second-order derivative of U_Q is, therefore, negative and so U_Q is globally concave for $e_p>0$, $e_Q>0$. Further, the first-order conditions (A.1) and (A.2) being strictly positive at $(e_P=0,e_Q=e^c)$ and $(e_P=e^c,e_Q=0)$, respectively, implying that the solution e^c is indeed a global maxima for each player, given the other player plays e^c. □

Proof of Lemma 2. It follows from Lemma A.3 that $G(\theta)=\pi^e-e^c-k$ is strictly increasing in θ; and the continuity of $G(\theta)$ implies that we can find a threshold $\underline{\theta}$, which is increasing in k, such that $G(\theta)\geq 0$ if and only if $\theta\geq\underline{\theta}$. Further, $G(0)=-k$, and therefore, $\underline{\theta}=0$ if $k=0$. Similarly, Lemma A.2 finds that $F(\theta)=\pi^e-(1-e^c-k)$ is increasing and its continuity with respect to θ implies the existence of a threshold $\overline{\theta}$ such that $F(\theta)\leq 0$ if and only if $\theta\leq\overline{\theta}$. Further, $F(1)=k$, and therefore, $\overline{\theta}=1$ if $k=0$. □

Proof of Lemma 3. Because $R=2k$ at $\theta=0,1$, and because of the concavity of e^c by Assumption 1, $R>2k$ for all $\theta\in(0,1)$ and $\theta^R\in(0,1)$ uniquely solves $dR/d\theta=0$, which is equivalent to $de^c/d\theta=0$ and can be expressed as (17).

Further, because of the concavity of e^c, $\theta^R<1/2$ if the left-hand-side of (17) is strictly negative at $\theta=1/2$, and $\theta^R>1/2$ if the left-hand-side of 17 is strictly positive at $\theta=1/2$. The left-hand-side of (17), when computed at $\theta = 1/2$, is given by

$$
\begin{aligned}
&\frac{8q_{h1}h^{\mu}\left(h^{\mu}+1\right)}{\left(h^{\mu}+1\right)^{3}}\left(\frac{1}{h^{\mu}+1}-\frac{1}{2}\right)+\frac{8q_{1h}h^{\mu}\left(h^{\mu}+1\right)}{\left(h^{\mu}+1\right)^{3}}\left(\frac{h^{\mu}}{h^{\mu}+1}-\frac{1}{2}\right) \\
=&\frac{4q_{h1}h^{\mu}\left(1-h^{\mu}\right)}{\left(h^{\mu}+1\right)^{3}}+\frac{8q_{1h}h^{\mu}\left(h^{\mu}-1\right)}{\left(h^{\mu}+1\right)^{3}} \\
=&-\frac{4h^{\mu}\left(h^{\mu}-1\right)}{\left(h^{\mu}+1\right)^{3}}\left[q_{h1}-q_{1h}\right] \\
=&-\frac{16h^{\mu}\left(h^{\mu}-1\right)}{\left(h^{\mu}+1\right)^{3}}\left(\gamma-\frac{1}{2}\right)\left(m-\frac{1}{2}\right),
\end{aligned}
$$

which is strictly negative if $m > 1/2$, and is strictly positive if $m < 1/2$. Therefore, $\theta^R < 1/2$ if $m > 1/2$, and $\theta^R > 1/2$ if $m < 1/2$. Finally, if $m = 1/2$, then $\theta = 1/2$ is a solution of (17), implying $\theta^R = 1/2$. □

Proof of Proposition 3. We let θ_{PV} and θ_{NP} denote the priors under the presumption of validity and the presumption of no validity, respectively. Therefore, $\theta_{PV} = \alpha + (1-\alpha)m$ and $\theta_{NP} = m$. We let R_{PV} and R_{NP} denote the value of R under the presumption of validity and the presumption of no validity, respectively. Specifically, $R_{PV} = R(\theta_{PV})$ and $R_{NP} = R(\theta_{NP})$. Note that $R(0) = R(1) = 2k$, and by Assumption 1, R is concave in θ with a maximum at θ^R.

First, consider $m \geq 1/2$. Then, by Lemma 3, $\theta^R \leq 1/2$ and by concavity, θ^R is decreasing in $\theta \in \left[\theta^R, 1\right]$. Further, as $\theta_{PV} > \theta_{NP} = m \geq \theta^R$, we have $R(\theta_{PV}) < R(\theta_{NP})$. Hence, for all $m \geq 1/2$, $R_{PV} < R_{NP}$.

Next, we consider $m < 1/2$. Then, $\theta^R > 1/2$ and therefore, $m < \theta^R$. As $R(\theta)$ is concave with a unique maximum at θ^R, it follows that for every $\theta < \theta^R$, there exists some $f(\theta) \in \left[\theta^R, 1\right]$ such that $R(\theta) = R(f(\theta))$, $R(\theta) < R(\theta')$ for all $\theta' \in (\theta, f(\theta))$ and $R(\theta) > R(\theta')$ for all $\theta' \in (f(\theta), 1]$. Further, observe that the mapping $f(\theta)$ is decreasing in $\theta < \theta^R$, which follows from the fact that $R(\theta)$ is decreasing in the range $\left[\theta^R, 1\right]$.

Now, consider some $m < 1/2$ such that $R_{NP}(m) = R(\theta_{NP}(m)) < R(\theta_{PV}(m)) = R_{PV}(m)$, which is equivalent to $m < \theta_{PV}(m) < f(m)$ by construction of f. Then, for all $m' < m$, we have $\theta_{PV}(m') < \theta_{PV}(m)$ because θ_{PV} is increasing in m, and $f(m') > f(m)$ because f is decreasing in $m < \theta^R$. Together, it follows that $\theta_{PV}(m') < \theta_{PV}(m) < f(m) < f(m')$ and consequently, $R_{NP}(m') = R(\theta_{NP}(m')) < R(\theta_{PV}(m')) = R_{PV}(m')$. Next, consider some $m < 1/2$ such that $R_{NP}(m) > R_{PV}(m)$, which is equivalent to $f(m) < \theta_{PV}(m)$ by construction of f. Then, for all $m' \in (m, 1/2)$, we have $\theta_{PV}(m) < \theta_{PV}(m')$ because θ_{PV} is increasing in m, and $f(m') < f(m)$ because f is decreasing in $m < \theta^R$. Together, it follows that $f(m') < f(m) < \theta_{PV}(m) < \theta_{PV}(m')$ and consequently, $R_{PV}(m') = R(\theta_{PV}(m')) < R(\theta_{NP}(m')) = R_{NP}(m')$.

We have just shown that for any $m < 1/2$, if $R_{NP}(m) < R_{PV}(m)$, then for all $m' < m$, $R_{NP}(m') < R_{PV}(m')$, and if $R_{PV}(m) < R_{NP}(m)$, then for all $m' > m$, $R_{PV}(m') < R_{NP}(m')$. This observation, together with the fact $R_{PV}|_{m=1/2} < R_{NP}|_{m=1/2}$, implies that $R_{NP} < R_{PV}$ if and only if m is less than some threshold and the threshold is less than $1/2$. We refer to this threshold by m^R_{PV}. □

Proof of Lemma 4. To prove part (i), consider the derivative of E with respect to θ:

$$\frac{dE(\theta)}{d\theta} = \frac{q_1 h^\mu}{((1-\theta)+\theta h^\mu)^2} + q_2 + \frac{q_3 h^\mu}{((1-\theta)h^\mu+\theta)^2},$$

where

$$\begin{aligned} q_1 =& (1-m)(1-\gamma)^2 - m\gamma^2 = (1-\gamma)^2 - m\left[(1-\gamma)^2+\gamma^2\right], \\ q_2 =& 2(1-2m)\gamma(1-\gamma), \\ q_3 =& (1-m)\gamma^2 - m(1-\gamma)^2 = \gamma^2 - m\left[(1-\gamma)^2+\gamma^2\right]. \end{aligned}$$

Observe that q_1, q_2, and q_3 are linear and decreasing in m. Further, as $1-\gamma < \gamma$, when $m < 1/2$, $q_2 > 0$, $q_3 > 0$, and for sufficiently low values of m, $q_1 > 0$. It follows that there is a threshold value $\underline{m} < 1/2$ such that for any $m \leq \underline{m}$, all q_1, q_2, and q_3 are positive and $dE(\theta)/d\theta > 0$ for all $\theta \in [0,1]$. Similarly, when $m > 1/2$, $q_2 < 0$, $q_1 < 0$, and for sufficiently high values of m, $q_3 < 0$. Therefore, there is a threshold value $\overline{m} > 1/2$ such that for any $m \geq \overline{m}$, all q_1, q_2, and q_3 are negative and $dE(\theta)/d\theta < 0$ for all $\theta \in [0,1]$. This completes the proof of part (i) of the lemma.

To prove part (ii), observe that $q_1 < q_3$ and

$$h^\mu/((1-\theta)+\theta h^\mu)^2 \lesseqqgtr h^\mu/((1-\theta)h^\mu+\theta)^2 \Leftrightarrow \frac{1}{2} \lesseqqgtr \theta.$$

Let us first consider $m \leq 1/2$ and $\theta \geq 1/2$. From $m \leq 1/2$, it follows that $q_2 \geq 0$ and $q_3 > 0$. If $q_1 \geq 0$, then it trivially follows that $dE(\theta)/d\theta \geq 0$. Consider the possibility that $q_1 < 0$. Further, $m \leq 1/2$ implies that $-q_3 < q_1$, or, equivalently, $q_1 + q_3 > 0$. From $\theta \geq 1/2$, it follows that $h^\mu/((1-\theta)+\theta h^\mu)^2 \leq h^\mu/((1-\theta)h^\mu+\theta)^2$. Applying these observations, we get

$$dE(\theta)/d\theta = \frac{q_1 h^\mu}{((1-\theta)+\theta h^\mu)^2} + q_2 + \frac{q_3 h^\mu}{((1-\theta)h^\mu+\theta)^2} \geq \frac{h^\mu(q_1+q_3)}{((1-\theta)+\theta h^\mu)^2} + q_2 \geq 0.$$

Next, consider $m \geq 1/2$ and $\theta \leq 1/2$. From $m \geq 1/2$, it follows that $q_2 \leq 0$, $q_1 < 0$. If $q_3 \leq 0$, it trivially follows that $dE^{LT}(\theta)/d\theta \leq 0$. Consider the possibility that $q_3 > 0$. Further, $m \geq 1/2$ implies that $q_1 < -q_3$, or, equivalently, $q_1 + q_3 < 0$. From $\theta \leq 1/2$, it follows that $h^\mu/((1-\theta)+\theta h^\mu)^2 \geq$

$h^{\mu}/\left((1-\theta)h^{\mu}+\theta\right)^2$. Applying these observations, we get q2

$$dE(\theta)/d\theta = \frac{q_1 h^{\mu}}{\left((1-\theta)+\theta h^{\mu}\right)^2} + q_2 + \frac{q_3 h^{\mu}}{\left((1-\theta)h^{\mu}+\theta\right)^2} \leq \frac{h^{\mu}(q_1+q_3)}{\left((1-\theta)+\theta h^{\mu}\right)^2} + q_2 \leq 0.$$

When $m = 1/2$, the analysis above suggests that $E(\theta)$ is increasing in θ for $\theta \geq 1/2$ and is decreasing in θ for $\theta \leq 1/2$, implying that $E(\theta)$ reaches its minimum at $\theta = 1/2$. This completes the proof of part (ii) of the lemma. □

Proof of Proposition 4. We let θ_{PV} and θ_{NP} denote the priors under the presumption of validity and the presumption of no validity, respectively. Therefore, $\theta_{PV} = \alpha + (1-\alpha)m$ and $\theta_{NP} = m$. Similarly, we let E_{PV} and E_{NP} denote the value of E under the presumption of validity and the presumption of no validity, respectively. Specifically, $E_{PV} = E(\theta_{PV})$ and $E_{NP} = E(\theta_{NP})$. To prove the part (i) of the proposition, we will show that $\Delta_{PV} := E_{PV} - E_{NP} \geq 0$ for all $m \leq 1/2$.

Consider the range of parameter values such that the litigation trial regime prevails in equilibrium in both the presumption scenarios, PV and NP. Therefore, using (20), $\Delta_{PV} = E(\theta_{PV}) - E(m)$ can be expressed as

$$\Delta_{PV} = h^{\mu}\alpha(1-m)\left[\frac{q_1}{ab} + \frac{q_3}{cd} + \frac{q_2}{h^{\mu}}\right],$$

where

$$q_1 = (1-\gamma)^2 - m\left(\gamma^2 + (1-\gamma)^2\right), q_2 = 2\gamma(1-\gamma)(1-2m), q_3 = \gamma^2 - m\left(\gamma^2+(1-\gamma)^2\right),$$
$$a = (1-\theta_{PV}) + \theta_{PV}h^{\mu}, b = (1-m)+mh^{\mu}, c = (1-m)h^{\mu}+m, d = (1-\theta_{PV})h^{\mu}+\theta_{PV}.$$

Observe that for $m \leq 1/2$, $q_2 \geq 0$ and $q_3 > 0$.

For $m \leq (1-\gamma)^2/\left(\gamma^2+(1-\gamma)^2\right) < 1/2$, we have $q_1 \geq 0$, and it therefore trivially follows that $\Delta_{PV} \geq 0$.

For $m \in \left((1-\gamma)^2/\left(\gamma^2+(1-\gamma)^2\right), 1/2\right]$, we have $q_1 < 0$, $q_2 \geq 0$, and $q_3 > 0$. In this range of values of m, $(q_1/ab) + (q_3/cd) \geq 0$ implies $\Delta_{PV} \geq 0$.

Claim 1: $(q_1/ab) + (q_3/cd) \geq 0$ at $m = 1/2$.

Proof of Claim 1: At $m = 1/2$, we have $a > b = (h^{\mu}+1)/2 = c > d$ and $q_3 = -q_1 = \gamma^2 - (1-\gamma)^2$. Therefore,

$$(q_1/ab) + (q_3/cd) = \frac{2\left(\gamma^2-(1-\gamma)^2\right)}{h^{\mu}+1}\left(\frac{1}{d}-\frac{1}{a}\right) > 0,$$

which proves Claim 1.

Claim 2: (q_1/ab) is decreasing in m for $m \in \left((1-\gamma)^2/\left(\gamma^2+(1-\gamma)^2\right), 1/2\right]$.

Proof of Claim 2: Consider the derivative of (q_1/ab) (for brevity, we write df/dm as f' when f is a function of m):.

$$\frac{d}{dm}\frac{q_1}{ab} = \frac{(q_1)'\,ab - q_1\,(ab)'}{(ab)^2} \leq 0 \Leftrightarrow (q_1)'\,ab \leq q_1\,(ab)' \Leftrightarrow \frac{(q_1)'}{q_1} \geq \frac{(ab)'}{ab}. \tag{A.4}$$

The inequality is reversed in the last part of the above chain of equivalent conditions because $q_1 < 0$. Note that

$$\frac{(q_1)'}{q_1} = \frac{\left(\gamma^2 + (1-\gamma)^2\right)}{m\left(\gamma^2 + (1-\gamma)^2\right) - (1-\gamma)^2} > 0$$

for $m \in \left((1-\gamma)^2 / \left(\gamma^2 + (1-\gamma)^2\right), 1/2\right]$ and is decreasing in m. Therefore, its minimum value is reached at $m = 1/2$ and

$$\min_{m \in \left(\frac{(1-\gamma)^2}{\gamma^2+(1-\gamma)^2}, 1/2\right]} \frac{(q_1)'}{q_1} = \frac{2\left(\gamma^2 + (1-\gamma)^2\right)}{\gamma^2 - (1-\gamma)^2} > 2. \tag{A.5}$$

Further,

$$\frac{(ab)'}{ab} = \frac{a'}{a} + \frac{b'}{b} = \frac{(1-\alpha)(h^\mu - 1)}{(1-\theta_{PV}) + \theta_{PV} h^\mu} + \frac{(h^\mu - 1)}{(1-m) + m h^\mu},$$

and each of the two terms is less than 1 by Assumption 1. Therefore,

$$\max_{m \in \left(\frac{(1-\gamma)^2}{\gamma^2+(1-\gamma)^2}, 1/2\right]} \frac{(ab)'}{ab} \leq 2. \tag{A.6}$$

Together (A.5) and (A.6) imply that (A.4) must hold for all $m \in \left((1-\gamma)^2 / \left(\gamma^2 + (1-\gamma)^2\right), 1/2\right]$, and therefore, (q_1/ab) is decreasing in this range of values of m, which completes the proof of Claim 2.

Claim 3: (q_3/cd) is decreasing in m for $m \in \left((1-\gamma)^2 / \left(\gamma^2 + (1-\gamma)^2\right), 1/2\right]$.

Proof of Claim 3: Consider the derivative of (q_3/cd):

$$\frac{d}{dm}\frac{q_3}{cd} = \frac{(q_3)'\,cd - q_3\,(cd)'}{(cd)^2} \leq 0$$

$$\Leftrightarrow (q_3)'\,cd \leq q_3\,(cd)' \Leftrightarrow \frac{(q_3)'}{q_3} \leq \frac{(cd)'}{cd} = \frac{c'}{c} + \frac{d'}{d}. \tag{A.7}$$

Observe that

$$\frac{(q_3)'}{q_3} = -\frac{\left(\gamma^2+(1-\gamma)^2\right)}{\gamma^2-m\left(\gamma^2+(1-\gamma)^2\right)},$$
$$\frac{c'}{c} = -\frac{(h^\mu-1)}{h^\mu-m(h^\mu-1)},$$
$$\frac{d'}{d} = -\frac{(1-\alpha)(h^\mu-1)}{h^\mu-\theta_{PV}(h^\mu-1)},$$

and, therefore, (A.7) can be rewritten as

$$\frac{(h^\mu-1)}{h^\mu-m(h^\mu-1)}+\frac{(1-\alpha)(h^\mu-1)}{h^\mu-\theta_{PV}(h^\mu-1)} \leq \frac{\left(\gamma^2+(1-\gamma)^2\right)}{\gamma^2-m\left(\gamma^2+(1-\gamma)^2\right)}. \tag{A.8}$$

Further, note that

$$\frac{(1-\alpha)(h^\mu-1)}{h^\mu-\theta_{PV}(h^\mu-1)} \leq \frac{(h^\mu-1)}{h^\mu-m(h^\mu-1)}$$
$$\Leftrightarrow (1-\alpha)h^\mu-(1-\alpha)m(h^\mu-1) \leq h^\mu-\theta_{PV}(h^\mu-1)$$
$$\Leftrightarrow \alpha(h^\mu-1) \leq \alpha h^\mu,$$

which holds true. Therefore, to prove (A.8), it is sufficient to show that

$$\frac{2(h^\mu-1)}{h^\mu-m(h^\mu-1)} \leq \frac{\left(\gamma^2+(1-\gamma)^2\right)}{\gamma^2-m\left(\gamma^2+(1-\gamma)^2\right)}.$$

We can simplify the above inequality as follows:

$$\frac{2(h^\mu-1)}{h^\mu-m(h^\mu-1)} \leq \frac{\left(\gamma^2+(1-\gamma)^2\right)}{\gamma^2-m\left(\gamma^2+(1-\gamma)^2\right)}$$
$$\Leftrightarrow 2(h^\mu-1)(1-m)\gamma^2-2(h^\mu-1)m(1-\gamma)^2 \leq \gamma^2(h^\mu-m(h^\mu-1))+(1-\gamma)^2(h^\mu-m(h^\mu-1))$$
$$\Leftrightarrow -(1-\gamma)^2(2(h^\mu-1)m+h^\mu-m(h^\mu-1)) \leq \gamma^2(h^\mu-m(h^\mu-1)-2(h^\mu-1)(1-m))$$
$$\Leftrightarrow -(1-\gamma)^2((h^\mu-1)m+h^\mu) \leq \gamma^2(2-h^\mu+m(h^\mu-1)),$$

which holds true, because the right-hand side is positive by Assumption 1 and the left-hand side is negative. This proves Claim 3.

Claims 1, 2, and 3 show that $(q_1/ab)+(q_3/cd)\geq 0$ at $m=1/2$ and $(q_1/ab)+(q_3/cd)$ is decreasing in m for $m\in\left((1-\gamma)^2/\left(\gamma^2+(1-\gamma)^2\right),1/2\right]$. Therefore,$(q_1/ab)+(q_3/cd)\geq 0$, and consequently, $\Delta_{PV}\geq 0$ for $m\in\left((1-\gamma)^2/\left(\gamma^2+(1-\gamma)^2\right),1/2\right]$. As we have already shown that $\Delta_{PV}\geq 0$ for $m\leq(1-\gamma)^2/\left(\gamma^2+(1-\gamma)^2\right)$, it follows that $\Delta_{PV}\geq 0$ for $m\leq 1/2$.

At $m=1$, $\theta_{PV}=m$, and $\Delta_{PV}=E^{LT}(\theta_{PV})-E^{LT}(m)=0$. Δ_{PV} can be negative for $m>1/2$; however, because of continuity, we can find a threshold $m^E_{PV}\geq 1/2$ such that $\Delta_{PV}\geq 0$ if $m\leq m^E_{PV}$. Further, note that the result provides only a sufficient condition, but not a necessary condition, because Δ_{PV} is not monotone in m. $\square$