An All-pay Auction with Endogenous Entry and Pre-contest Investment

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March 25, 2023

1 Extended Abstract

This article concerns a competition among agents in the absence of property rights, where investing in a technology can give an agent a competitive advantage. However, agents may be uncertain about which technology will be favored in future competitions or whether they succeed in making technological progress. Given such uncertainty, how should an agent decide whether to invest in a technology?

There are two types of investment involved in this problem. Firstly, in the absence of property rights, agents engage in an appropriation contest where they invest resources to outperform each other, driven by their rent-seeking incentives. The literature on rent-seeking incentives for investment is well established (Konrad 2009). However, this article primarily explores the second type of investment, which involves investing in technology for competitive advantage with uncertain returns. At the time of investment, agents are unaware of the favorability of any specific technology for future competition or their chances of achieving technological prowess, making returns to investment uncertain. This uncertainty can have significant implications. For example, large investments can lead to substantial returns from the future contest, which is socially beneficial. However, uncertainty makes investment less attractive and can result in inefficient investment. Furthermore, since the investment decision is endogenous, uncertainty about the number of investing agents can discourage new agents from entering the contest, leading to further loss of welfare. While the problem of investment under uncertainty is quite general, the following applications can best illustrate our research question.

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Consider, for example, the economics of cryptocurrency mining. Since the introduction of the first cryptocurrency *Bitcoin* by Nakamoto (2008), the total market capitalization (around 1 trillion USD as of March 2023) of cryptocurrencies has steadily increased over time and numerous cryptocurrencies, including Litecoin, Ethereum, Tether, BNB, XRP, Cardano, have emerged. Cryptocurrency mining involves introducing new blocks of a currency to its existing circulation. This process is underpinned by a protocol that allows cryptocurrencies to function as a peer-to-peer (P2P) decentralized network. Miners verify transactions between participants over the blockchain network and add them to the distributed public ledger. In exchange, miners receive a block reward in the form of new currency plus a transaction fee. Bitcoin miners, for instance, currently receive 6.25 bitcoin as a block reward.

Mining can be a profitable venture, but its profitability is subject to various factors. Miners compete against each other to solve a cryptographic puzzle, with the winner receiving the block reward. Solving a puzzle requires expensive resources, such as specialized hardware equipment and a considerable amount of electricity. The cost and efficiency of mining equipments can vary significantly. For example, a general-purpose central processing unit (CPU) may not be expensive, but it is inefficient in solving the puzzle. Given the current level of difficulty, utilizing a high-end CPU to find a new block would require hundreds of years, making it an impractical pursuit due to the associated electricity costs. Professional miners therefore invest in more efficient but expensive application-specific integrated circuits (ASICs), which can only mine one type of cryptocurrency. Despite the potential for profit, miners are confronted with uncertainty regarding the return on their investment in hardware equipment and the selection of mining center locations. At the time of investment, they are uncertain about the future market value of the specific cryptocurrency that the ASIC can mine, as well as the future price of electricity at the mining location. Those factors determine the expected prize value and the cost of effort at the time the contest takes place. The uncertainty can discourage investment and entry in the mining industry; it is not surprising that a small number of large entities control cryptocurrency mining, despite the currency being conceptualized as a decentralized mean for facilitating transactions.

Another application of investment for competitive advantage can be found in the context of developing technology-driven products such as alternative fuel vehicles. For instance, various potential technologies like hydrogen fuel cells, Sodium-ion batteries, Lithium-sulphur batteries, can not only replace the traditional fuel cars but also bring significant improvements to the currently dominant lithium-ion battery technology used in the electric cars. Investing early in one of these technologies could provide an investor with a substantial competitive advantage and the potential for a monopoly rent. However, it's uncertain which technology will become dominant in the market.

In this paper, we study agents investment incentive to gain competitive advantage when they later engage in an appropriation contest. Two key questions motivate our work. First, how does uncertainty affect investment for competitive advantage? Second, how does agents' investment incentive interact with their incentives to enter the competition? As pointed out above in the context of cryptocurrency, one puzzling observation is the high concentration of mining activities among a limited set of players. Limited entry can soften competition and may lead to efficiency loss.

We address the above questions by developing a simple model, which proceeds in three stages. The game starts with N players, who differ in terms of their marginal costs of investment, which is stochastically distributed. At the very first stage, agents decide whether or not to enter into competition. Next, upon entry, they decide whether to invest in acquiring a skill which can enable them to realize a larger prize value − an alternate way of interpreting skill could be a reduction in marginal costs of contest effort − which would create a favorable imbalance in future contest. The return to investment in skill acquisition is however uncertain − An agent acquires skill with some probability $q \in (0, 1)$. Finally, the set of entrants, some skilled and some unskilled, compete in an all-pay auction to win the prize.

We examine the equilibrium entry and investment strategies. We identify two marginal-costs thresholds that determine the entry and investment decisions. When these thresholds are strictly below their upper limits, entry and investment will be limited. We investigate how various parameters, including the extent of uncertainty in skill acquisition and the return from skill acquisition, affect the conditions for limited entry and investment. Interestingly, we discover a non-monotonic effect of the uncertainty in skill acquisition q. The non-monotonicity arises due to the dual effects that an increase in the likelihood of skill acquisition has. On one hand, it enhances an agent's competitive advantage in future contests. On the other hand, the agent also expects to encounter more skilled competitors, reducing both investment and entry incentives.