Alternate Setting with Mobility Allowed Along Conflict Path

1 Set-up and Analysis

To incorporate the possibility of inter-group mobility along the conflict path, we consider an alternative setting in which, agents can choose to switch groups both along the economic path, as well as on the path of conflict by incurring an individual cost of ϕ_d and ϕ_c respectively. We make the following adjustment to the original game. The economic path is unchanged in that after a sharing rule is accepted by the opposition, each individual can decide to switch groups by incurring a cost $\phi_d \in (0, 1)$.[1] Moreover, now, if the opposition rejects the sharing rule and wages conflict, each individual can still decide whether to remain in his own group or to switch to the other group, by incurring a cost, ϕ_c.

We allow the switching costs ϕ_c to be unrestricted, to get a full characterization. To keep the model consistent with the original one, we retain the assumption that individuals are myopic in the sense that the switching incentive (in both the conflict and economic paths) is driven solely by a desire to acquire resources in the current period. Recall that in our original model, if conflict is waged, then the incumbent group consumes k fraction of the resources where as the opposition group consumes nothing in the current period.

In the remainder of this section, we characterize equilibrium play, and state an analog of our main result in this setting. The proofs are contained in the appendix.

1.1 Equilibrium play in the second period

The second period results will be unaffected, as the opposition has no incentive to engage in conflict. In terms of our original model, Lemmata 1 and 2, and Proposition 1 remain unchanged.

1.2 Equilibrium play in the first period

Consider the temporal game in period 1. Without loss of generality, we assume that A is the incumbent group in period 1. We introduce new notations here: Let π_c^A and π_d^A denote A's post-switching group size at the end of period 1, along the conflict path and along the economic path respectively. We drop subscript 1 as this distinction, in any case, is relevant only in period 1.

1.2.1 Play along the economic path in period 1

The results of the original model, in particular, Lemmata 3 and 4, are unaffected.

1.2.2 Play along the conflict path in period 1

Given the group size distribution $\left(\pi_0^A, 1 - \pi_0^A\right)$ and in the absence of any switching, members of group A get $\frac{k}{\pi_0}$ and members of group B get zero. So, members of group B have an incentive to switch if

[1] Recall, that we restricted ϕ_d to lie in the range $(0, 1)$ for reasons of tractability. The assumption ensures that the optimal allocation rule along the economic path lies in the open interval (0,1).

$\frac{k}{\pi_0} > \phi_C$. The resulting group size of A at the end of period 1 along the conflict path is given by

$$\pi_c^A = \begin{cases} 1 & \text{if } \phi_c \leq k \\ \frac{k}{\phi_c} & \text{if } k < \phi_c \leq \frac{k}{\pi_0^A} \\ \pi_0^A & \text{if } \frac{k}{\pi_0^A} < \phi_c \end{cases}.$$

To see this, note that if ϕ_c is less than k, even after everyone switches from B to A, the switching members payoff exceeds the switching cost. If ϕ_c is above k but below $\frac{k}{\pi_0^A}$, only a fraction of B members switching, and the resulting size must solve that $\frac{k}{\pi_c^A} = \phi_c$. Finally, when ϕ_c is above $\frac{k}{\pi_0^A}$, members of B have no incentive to switch.

We can now derive expressions for the payoffs along the economic and conflict paths, respectively.

$$\begin{aligned}
E_A(\alpha_1^A, \pi_d^A) &= \frac{\alpha_1^A}{\pi_d^A} + p_d(\pi_d^A)[1 + \phi_d(1 - \pi_d^A)] + [1 - p_d(\pi_d^A)][1 - \phi_d(1 - \pi_d^A)] \\
&= \frac{\alpha_1^A}{\pi_d^A} + 1 + \phi_d(1 - \pi_d^A)[2p_d(\pi_d^A) - 1] \\
E_B(\alpha_1^A, \pi_d^A) &= \frac{1-\alpha_1^A}{1-\pi_d^A} + p_d(\pi_d^A)[1 - \phi_d\pi_d^A] + [1 - p_d(\pi_d^A)][1 + \phi_d\pi_d^A] \\
&= \frac{1-\alpha_1^A}{1-\pi_d^A} + 1 + \phi_d\pi_d^A[1 - 2p_d(\pi_d^A)] \\
P_A &= \frac{k}{\pi_c^A} + p_c(\pi_c^A)[1 + \phi_d(1 - \pi_c^A)] + [1 - p_c(\pi_c^A)][1 - \phi_d(1 - \pi_c^A)] \\
&= \frac{k}{\pi_c^A} + 1 + \phi_d(1 - \pi_c^A)(2p_c(\pi_c^A) - 1) \\
P_B &= p_c(\pi_c^A)[1 - \phi_d\pi_c^A] + [1 - p_c(\pi_c^A)][1 + \phi_d\pi_c^A] \\
&= 1 + \phi_d\pi_c^A(1 - 2p_c(\pi_c^A))
\end{aligned}$$

Note that these payoff expressions only involve ϕ_d because, in the second period, play will proceed along the economic path in equilibrium. The effect of ϕ_c on the payoff expressions is indirect $-$ It is present through its effect on the first period group size π_c^A.

1.2.3 Opposition's preference for conflict in period 1

The following lemma shows that the opposition accepts a sharing rule if and only if the incumbent's share of the surplus is below a threshold. The threshold is increasing in the mobility cost along the conflict path.

Lemma A.1. *Assume that A is the incumbent group in period 1 with size π_0^A.*

1. *There is a threshold $\bar{\alpha} \in [0, 1]$ such that group B accepts an allocation α_1^A, proposed by group A, if and only if the allocation satisfies $\alpha_1^A \leq \bar{\alpha}$.*
2. *There exists a threshold $\phi_1(\phi_c) > 0$ such that $\bar{\alpha} = 1$ if $\phi_d \leq \phi_1(\phi_c)$. Thus, all allocations are accepted if $\phi_d < \phi_1(\phi_c)$.*

The proof is given in the appendix.

1.2.4 Incumbent's preference for conflict in period 1

Lemma A.1 tells us that $E := [0, \bar{\alpha}]$ is the set of allocations that induces the opposition to follow the economic path, and the complement (which we denote by P) is the set of allocations that induces the opposition to engage in conflict. To understand which path of play the incumbent would prefer, we need to compare P_A with $\max_{\alpha_1^A \in E} E_A(\alpha_1^A)$. We show in the following lemma that there is a threshold such that the incumbent's maximal payoff on the economic path is higher than that on the conflict path if and only if the cost of mobility is above the threshold.

Lemma A.2. $\max_{\alpha_1^A \in E} E_A(\alpha_1^A) \geq P_A \Leftrightarrow \phi_d \geq \phi_2(\phi_c)$*. In addition, we have* $\phi_2(\phi_c) \geq 0$ if and only if $1 \leq \min\left\{\phi_c, \frac{k}{\pi_0}\right\}$.

The proof of Lemma A.2 is in the Appendix. It remains to characterize the equilibrium for $\phi_d > \max\{\phi_1(\phi_c), \phi_2(\phi_c)\}$. In this range, the incumbent prefers the economic path, and it's most preferred allocation choice is α^e. Next, we characterize the conditions under which the opposition does, indeed, accept α^e. We show in the lemma below, that there is a threshold $\phi_3(\phi_c)$, above which α^e is not implementable along the economic path.

Lemma A.3. *Assume that A is the incumbent group in period 1 with size π_0^A. There exists a threshold $\phi_3(\phi_c) > 0$ such that*

1. *Group B accepts allocation α^e if and only if the cost of mobility ϕ_d is weakly less than the threshold $\phi_3(\phi_c)$.*
2. *If $\phi_d > \phi_3(\phi_c)$, the maximum share that group A can retain, while still inducing the economic path, is $\bar{\alpha}$, where $\bar{\alpha} < \alpha^e$.*

The lemma implies that if $\phi_d > \phi_3(\phi_c)$, then the incumbent must choose between inducing the economic path (by offering $\bar{\alpha}$) and inducing conflict. We can now characterize equilibrium play in period 1.

Proposition A.1. *Assume that A is the incumbent group in period 1 with size π_0^A. The equilibrium regimes (and respective allocations α_1^*) that arise in period 1, are characterized as follows:*

- *If $\phi_d \leq \phi_1(\phi_c)$, then the no-conflict regime prevails (with equilibrium allocation $\alpha_1^* = \alpha^e$).*
- *If $\phi_d \in (\phi_1(\phi_c), \phi_2(\phi_c)]$, then the open-conflict regime occurs (with $\alpha_1^* = 1$).*
- *If $\phi_d \in (\max\{\phi_1(\phi_c), \phi_2(\phi_c)\}, \phi_3(\phi_c)]$, then the no-conflict regime prevails (with $\alpha_1^* = \alpha^e$).*
- *If $\phi_d > \max\{\phi_2(\phi_c), \phi_3(\phi_c)\}$ then either peaceful belligerence regime or open conflict regime exists.*

1.3 Discussion

The thresholds $\phi_1(\phi_c)$, $\phi_2(\phi_c)$ and $\phi_3(\phi_c)$ are explicitly derived in the proofs below, and it is easy to see that they are all continuous functions of ϕ_c. While it can be shown that $\phi_1(\phi_c)$ is always less than $\phi_3(\phi_c)$, comparison of $\phi_2(\phi_c)$ with $\phi_1(\phi_c)$ or with $\phi_3(\phi_c)$ can go either way. Comparing the expressions of $\phi_1(\phi_c)$ and $\phi_2(\phi_c)$, we can see that open conflict can emerge at an intermediate range of mobility cost ($\phi_1(\phi_d) < \phi_d < \phi_2(\phi_c)$) in the economic path when the mobility cost in conflict is sufficiently high (as $\phi_2(\phi_c)$ is increasing in ϕ_c) and k is sufficiently higher than π_0. Specifically, a necessary condition is that $1 \leq \min\left\{\phi_c, \frac{k}{\pi_0^A}\right\}$. Comparing the value ϕ_c with $\frac{k}{\pi_0^A}$, two cases are possible.

Case 1: $\phi_1(\phi_d) < \phi_d < \phi_2(\phi_c)$ and $1 \ll \frac{k}{\pi_0^A} \leq \phi_c$. In this case open conflict emerges in equilibrium and we do not see any switching in the conflict path.

Case 2: $\phi_1(\phi_d) < \phi_d < \phi_2(\phi_c)$ and $1 \ll \phi_c < \frac{k}{\pi_0^A}$. In this case open conflict emerges in equilibrium and we see partial switching in the conflict path.

The second case illustrates a situation when it is possible to observe open conflict in equilibrium even when mobility occurs along the conflict path. Using the continuity property of the thresholds and an argument similar to that in Corollary 1 in the main paper, we can show that both Case 1 and Case 2 above can indeed arise for a wide range of parameter values. In this sense, our equilibrium characterization does not depend on the assumption of "no switching during conflict."

Appendix: Proofs

Proof of Lemma A.1:

Proof. We compare B's payoff along the economic path and conflict path. From the proof of Lemma 4 of the original model, we already know that $E_B\left(\alpha_1^A, \pi_d^A(\alpha_1^A)\right)$ first increases, then decreases and

$E_B\left(0, \pi_d^A(0)\right) > P_B$. As P_B is independent of α_1^A, $E_B\left(\alpha_1^A, \pi_d^A(\alpha_1^A)\right)$ can therefore only intersect P_B from above. We define $\overline{\alpha}$ as the value of α_1^A at which E_B intersects P_B. If $E_B\left(\alpha_1^A, \pi_d^A(\alpha_1^A)\right)$ lies above P_B for all $\alpha_1^A \in [0,1]$, we set $\overline{\alpha} = 1$. By construction, it is easy to see that for all $\alpha_1^A \leq \overline{\alpha}$, $E_B\left(\alpha_1^A, \pi_d^A(\alpha_1^A)\right) \geq P_B$, and for all $\alpha_1^A > \overline{\alpha}$, $E_B\left(\alpha_1^A, \pi_d^A(\alpha_1^A)\right) < P_B$. Therefore B accepts an allocations α_1^A if and only if $\alpha_1^A \leq \overline{\alpha}$.

Note that $\overline{\alpha} = 1$ if and only if $E_B(1,1) \geq P_B$. As $E_B(1,1)$ is not well-defined, we compare the payoffs in limit.

$$\lim_{\alpha_1^A \to 1} E_B\left(\alpha_1^A, \pi_d^A(\alpha_1^A)\right) = 2(1 - \phi_d p_d(1))$$

Comparing the above expression with P_B, we get

$$\begin{aligned} E_B(1,1) &\geq P_B \\ \text{(after rearranging terms)} & \\ \Leftrightarrow p_d(1) &\leq \frac{1}{2}\left[\frac{1}{\phi_d} - \pi_c^A(1 - 2p_c(\pi_c^A))\right] \triangleq G(\phi_d, \phi_c) \\ \Leftrightarrow \phi_d &\leq \frac{1}{2p_d(1) + \pi_c^A(1 - 2p_c(\pi_c^A))} \triangleq \phi_1(\phi_c). \end{aligned}$$

Together, we can characterize $\overline{\alpha}$ as follows:

If $p_d(1) \leq G(\phi_d, \phi_c)$, or equivalently if $\phi_d \leq \phi_1(\phi_c)$, we have $\overline{\alpha} = 1$.

If $\pi_0^A p_d(\pi_0^A) \leq G(\phi_d, \phi_c) < p_d(1)$, then $\overline{\alpha}$ solves the equation $\pi_d^A\left(\alpha_1^A\right) p_d(\pi_d^A\left(\alpha_1^A\right)) = G(\phi_d, \phi_c)$.

If $G(\phi_d, \phi_c) < \pi_0^A p_d(\pi_0^A)$, then $\overline{\alpha}$ solves the equation $\frac{1-\alpha_1^A}{1-\pi_0^A} + 1 + \phi_d \pi_0^A[1 - 2p_d(\pi_0^A)] = 1 + \phi_d \pi_c^A(1 - 2p_c(\pi_c^A))$. □

Proof of Lemma A.2:

Proof. We compare $E_A\left(\alpha_1^e, \pi_d^A(\alpha_1^e)\right)$ with P_A. Notice that $\alpha_1^e = f(\overline{\pi}^A) = \overline{\pi}^A + \phi\overline{\pi}^A(1 - \overline{\pi}^A)$ from Lemma 4 of the original model. Thus, $E_A\left(\alpha_1^e, \pi_1^A(\alpha_1^e)\right) = 2 + 2\phi_d p_d(\overline{\pi}^A)(1 - \overline{\pi}^A)$.

$$\begin{aligned} E_A\left(\alpha_1^e, \pi_1^A(\alpha_1^e)\right) &\geq P_A \\ \Leftrightarrow 2 + 2\phi_d p_d(\overline{\pi}^A)(1 - \overline{\pi}^A) &\geq \frac{k}{\pi_c^A} + 1 + \phi_d(1 - \pi_c^A)(2p_c(\pi_c^A) - 1) \\ \Leftrightarrow \phi_d &\geq \left[\frac{\left(\frac{k}{\pi_c^A} - 1\right)}{\left(2p_d(\overline{\pi}^A)\left(1 - \overline{\pi}^A\right) + (1 - \pi_c^A)(1 - 2p_c(\pi_c^A))\right)}\right] \triangleq \phi_2(\phi_c). \end{aligned}$$

Note that the denominator is always positive, and so ϕ_2 is positive if and only if $\frac{k}{\pi_c^A} \geq 1$.

Based on the value of ϕ_c, we can write $\phi_2(\phi_c)$ as follows:

$$\phi_2(\phi_c) = \begin{cases} \frac{(k-1)}{2p_d(\overline{\pi}^A)\left(1-\overline{\pi}^A\right)} & \text{if } \phi_c \leq k \\ \frac{(\phi_c - 1)}{2p_d(\overline{\pi}^A)\left(1-\overline{\pi}^A\right)+(1-\pi_c^A)(1-2p_c(\pi_c^A))} & \text{if } k < \phi_c \leq \frac{k}{\pi_0} \\ \frac{\left(\frac{k}{\pi_0^A} - 1\right)}{2p_d(\overline{\pi}^A)\left(1-\overline{\pi}^A\right)+\left(1-\pi_0^A\right)\left(1-2p_c(\pi_0^A)\right)} & \text{if } \frac{k}{\pi_0} < \phi_c \end{cases}$$

It is easy to see that $\phi_2(\phi_c) \geq 0$ if and only if $1 \leq \min\left\{\phi_c, \frac{k}{\pi_0}\right\}$. □

Proof of Lemma A.3:

Proof. We compare $E_B\left(\alpha_1^e, \pi_d^A(\alpha_1^e)\right)$ with P_B. Notice that $\alpha_1^e = f(\overline{\pi}^A) = \overline{\pi}^A + \phi\overline{\pi}^A(1-\overline{\pi}^A)$ from Lemma 4 of the original model. Thus, $E_B\left(\alpha_1^e, \pi_1^A(\alpha_1^e)\right) = 2 - 2\phi_d\overline{\pi}^A p_d(\overline{\pi}^A)$.

$$\begin{aligned} E_B(\alpha^e, \pi_d^A(\alpha^e, \pi_0^A)) &\geq P_B \\ 2 - 2\phi_d\overline{\pi}^A p_d(\overline{\pi}^A) &\geq 1 + \phi_d \pi_c^A(1 - 2p_c(\pi_c^A)) \\ 1 &\geq \phi_d\left[2\overline{\pi}^A p_d(\overline{\pi}^A) + \pi_c^A(1-2p_c(\pi_c^A))\right] \end{aligned}$$

Define

$$\phi_3(\phi_c) = \begin{cases} \frac{1}{2\overline{\pi}^A p_d(\overline{\pi}^A) + \pi_c^A(1-2p_c(\pi_c^A))} & \text{if } \overline{\pi}^A p_d(\overline{\pi}^A) > \pi_c^A(p_c(\pi_c^A) - \frac{1}{2}) \\ \infty & \text{otherwise} \end{cases}.$$

We can therefore conclude that have $\alpha^e \in E$ if and only if $\phi_d \leq \phi_3(\phi_c)$. □

Proof of Proposition A.1:

Proof. The first three parts of the proposition directly follow from Lemma A.1, A.2 and A.3. By Lemma A.3, we know that if $\phi_d > \phi_3(\phi_c)$, the incumbent can at most keep $\overline{\alpha}$ while still inducing the economic path. The incumbent therefore chooses $\overline{\alpha}$ if and only if $E_A(\overline{\alpha}, \pi_d^A(\overline{\alpha}, \pi_0^A)) - P_A \geq 0$. Note that when $\overline{\alpha} < 1$, $\overline{\alpha}$ solves the equation

$$\begin{aligned} E_B(\alpha_1^A, \pi_d^A) &= P_B \\ \frac{1-\alpha_1^A}{1-\pi_d^A} + 1 + \phi_d\pi_d^A[1 - 2p_d(\pi_d^A)] &= 1 + \phi_d\pi_c^A(1-2p_c(\pi_c^A)) \\ \frac{\alpha_1^A}{\pi_d^A} + \phi_d\left(1-\pi_d^A\right)[2p_d(\pi_d^A) - 1] &= 1 + \frac{1}{\pi_d^A} + \phi_d\frac{\pi_c^A}{\pi_d^A}\left(1-\pi_d^A\right)(2p_c(\pi_c^A) - 1) \end{aligned}$$

Comparing $E_A(\overline{\alpha}, \pi_d^A(\overline{\alpha}, \pi_0^A))$ with P_A, we get

$$\begin{aligned} E_A(\overline{\alpha}, \pi_d^A(\overline{\alpha}, \pi_0^A)) &\geq P_A \\ \Leftrightarrow \frac{\overline{\alpha}}{\pi_d^A} + 1 + \phi_d(1-\pi_d^A)[2p_d(\pi_d^A) - 1] &\geq \frac{k}{\pi_c^A} + 1 + \phi_d(1-\pi_c^A)(2p_c(\pi_c^A) - 1) \\ \Leftrightarrow 1 + \frac{1}{\pi_d^A} + \phi_d\frac{\pi_c^A}{\pi_d^A}\left(1-\pi_d^A\right)(2p_c(\pi_c^A) - 1) &\geq 1 + \frac{k}{\pi_c^A} + \phi_d(1-\pi_c^A)(2p_c(\pi_c^A) - 1) \end{aligned}$$

The comparison between $E_A(\overline{\alpha}, \pi_d^A(\overline{\alpha}, \pi_0^A))$ and P_A can go either way. To illustrate, consider the following two cases

First, consider, ϕ_c in $\left(k, \frac{k}{\pi_0}\right]$ such that $\pi_d^A = \pi_c^A$ (it is always possible to choose such a ϕ_c as π_c^A moves between π_0 and 1, and $\pi_d^A \in [\pi_0, 1]$). In this case, $E_A - P_A = \frac{1-k}{\pi_c^A}$, which is always positive. In this case, we observe peaceful belligerence.

Second, note that for any given k, P_A can be made arbitrarily large by taking small π_0 and large ϕ_c (when $\phi_c\left(k, \frac{k}{\pi_0}\right]$). For a given ϕ_d, E_A is bounded above by $1 + \frac{1}{\pi_d^A} + \phi_d\left(\frac{1}{\pi_d^A} - 1\right)$. Therefore, P_A can exceed E_A when π_0 is small, ϕ_c is large and π_d^A is sufficiently larger than π_0. In this case, we observe open conflict. □