Online Appendix for Competitive Balance: Information Disclosure and Discrimination in an Asymmetric Contest

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Appendix B

Appendix B contains supplementary materials for Clark and Kundu, "Competitive Balance: Information Disclosure and Discrimination in an Asymmetric Contest."

We generalize the state space to include values that are not reciprocal to each other. In particular, $S := \{x, y\}, 0 < x < 1 < y$, and $\Pr[s = x] = q \in (0, 1)$. The analysis begins at stage 4. The contest can take place under two possible information structures: i) full information and ii) asymmetric information.

B.1 Full-information contest

The analysis of the full information contest remains the same as done in the basic model. In particular, the principal's ex ante expected payoff under full information disclosure is

$$V_P^F(\alpha, q) = \mathbb{E}_q \left[\frac{2\alpha s}{(1 + \alpha s)^2} \right]. \tag{B.1}$$

B.2 Asymmetric-information contest

The information disclosure policy (chosen at stage 2) causes I to update his belief about the distribution over the state space and the updated belief affects the contest effort under asymmetric information. Denote I's updated belief by $p \in [0, 1]$, where $p = \Pr[s = x]$. As

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in our basic model, two possibilities may arise, one in which both skill types are active, and one in which the low ability newcomer does not exert effort in the contest. The following lemma documents the principal's expected payoff in the asymmetric-information contest:

Lemma B.1. *Let $\eta(x,y) := \frac{\sqrt{y}-\sqrt{x}}{y\sqrt{x}}$ and $\tilde{q}(x,y) := 1-\frac{\alpha}{\eta(x,y)}$. In the Bayes-Nash equilibrium of the asymmetric information contest, the principal's expected payoff is*

$$V_P^A(\alpha,p) = \begin{cases} 2\alpha\left(\frac{\mathbb{E}_p\left[\frac{1}{\sqrt{s}}\right]}{\alpha+\mathbb{E}_p\left[\frac{1}{s}\right]}\right)^2 & \text{if } \{\alpha > \eta(x,y)\} \cup \{\{\alpha < \eta(x,y)\} \cap \{p \in (\tilde{q}(x,y),1]\}\} \\ 2\alpha y\left(\frac{1-p}{\alpha y+(1-p)}\right)^2 & \text{if } \{\alpha \leq \eta(x,y)\} \cap \{p \in [0,\tilde{q}(x,y)]\} \end{cases} \tag{B.2}$$

Proof. The optimal effort of N will be a function of the true value of s, whilst the effort of I will be conditioned upon his belief. Assuming an interior solution, we can derive the best response of each agent from the respective first-order conditions. In particular, for given e_I^A, $e_N^A(s)$ solves

$$\frac{\alpha s e_I^A}{\left(\alpha s e_N^A(s)+e_I^A\right)^2} = 1, \tag{B.3}$$

and for a given profile $e_N^A(s), s \in S$, e_I^A solves

$$\mathbb{E}_p\left[\frac{\alpha s e_N^A(s)}{\left(\alpha s e_N^A(s)+e_I^A\right)^2}\right] = 1. \tag{B.4}$$

Solving (B.3) and (B.4) together yield the equilibrium efforts

$$e_N^A(s) = \frac{\sqrt{\alpha s e_I^A}-e_I^A}{\alpha s}, \tag{B.5}$$

and

$$e_I^A = \alpha\left(\frac{\mathbb{E}_p\left[\frac{1}{\sqrt{s}}\right]}{\alpha+\mathbb{E}_p\left[\frac{1}{s}\right]}\right)^2. \tag{B.6}$$

Further, the fact that $e_I^A = \mathbb{E}_p\left[e_N^A(s)\right]$, which follows from (B.3) and (B.4), allows us to express the principal's expected payoff, expressed as a function of α and p, as

$$V_P^A(\alpha,p) = \mathbb{E}_p\left[e_N^A(s)\right] + e_I^A = 2\alpha\left(\frac{\mathbb{E}_p\left[\frac{1}{\sqrt{s}}\right]}{\alpha+\mathbb{E}_p\left[\frac{1}{s}\right]}\right)^2. \tag{B.7}$$

This solution is valid if we have an interior solution, which holds true as long as the right-hand side in (B.5) is strictly positive for both types. Using (B.6) in (B.5) for $s = x$, this is equivalent to

$$\frac{\mathbb{E}_p\left[\frac{1}{\sqrt{s}}\right]}{\alpha+\mathbb{E}_p\left[\frac{1}{s}\right]}<\sqrt{x}\Leftrightarrow\alpha>(1-p)\,\eta(x,y) \tag{B.8}$$

where $\eta(x,y)=\frac{\sqrt{y}-\sqrt{x}}{y\sqrt{x}}$. (B.8) holds for any p if $\alpha>\eta$, and otherwise for $\eta(x,y)>\alpha$ and $p>1-\frac{\alpha}{\eta(x,y)}=\tilde{q}(x,y)$. When $\alpha\leq\eta(x,y)$ and $p\in[0,\tilde{q}(x,y))$, $e_N^A(x)=0$. Then (B.3) and (B.4) imply $e_I^A=(1-p)\,e_N^A(y)$, which inserted into (B.3) gives

$$e_N^A(y)=\frac{\alpha y(1-p)}{(\alpha y+(1-p))^2}.$$

The principal's payoff is then

$$V_P^A(\alpha,p)=e_I^A+(1-p)\,e_N^A(y)=2(1-p)\,e_N^A(y)=2\alpha y\left(\frac{1-p}{\alpha y+(1-p)}\right)^2.$$

□

Note that $\eta(x,y)$ can be above 1 for low values of x (for any given y). Therefore, unlike our basic model, the low ability newcomer (of type x) may not exert effort even when the discrimination parameter α is above 1.

B.3 Information disclosure

The analyses at stage 3 and at stage 2 remain the same as in our basic model. In particular, the principal's expected payoff following the optimal information disclosure policy is $Cav(\alpha,q)$, the concave closure of $V_P^A(\alpha,q)$. Also, recall that $Cav(\alpha,q)$ is the indirect value function of the following optimization problem:

$$\max_{\{q_m\in[0,1],\beta_m\in[0,1]\}_{m\in M}}\sum_{m\in M}\beta_m V_P^A(\alpha,q_m) \tag{B.9}$$
$$\text{subject to}\sum_{m\in M}\beta_m=1\text{ and }\sum_{m\in M}\beta_m q_m=q.$$

We study the principal's optimal information disclosure policy from the shape of $V_P^A(\alpha,q)$ with respect to q, which is summed up in the following Lemma.

Lemma B.2. *Define* $\underline{\alpha}(x,y):=\frac{3}{\sqrt{xy}}+\frac{2}{y}$ *and* $\bar{\alpha}(x,y):=\frac{3}{\sqrt{xy}}+\frac{2}{x}$*. The following characterizes the shape of* $V_P^A(\alpha,q)$*:*

(a) *If* $0<\alpha<\frac{1}{\sqrt{xy}}$*, then* $V_P^A(\alpha,q)$ *is decreasing in* $q\in(0,1)$*. Further, for* $\min\left\{\frac{2}{y},\eta(x,y)\right\}<\alpha<\frac{1}{\sqrt{xy}}$*,* $V_P^A(\alpha,q)$ *is convex in* $q\in(0,1)$*; for* $0<\alpha\leq\min\left\{\frac{2}{y},\eta(x,y)\right\}<\frac{1}{\sqrt{xy}}$*,*

$V_P^A(\alpha,q)$ *is concave for* $q \in (0, min\{\tilde{q}(x,y), q'(x,y)\})$, *and piecewise convex for* $q \in (min\{\tilde{q}(x,y), q'(x,y)\}, 1)$ *over the two segments* $(min\{\tilde{q}(x,y), q'(x,y)\}, \tilde{q}(x,y))$ *and* $(\tilde{q}(x,y), 1)$, *where* $\tilde{q}(x,y) = 1 - \frac{\alpha}{\eta(x,y)}$, *and* $q'(x,y) = 1 - \frac{\alpha y}{2}$.

(b) *If* $\alpha = \frac{1}{\sqrt{xy}}$, *then* $V_P^A(\alpha,q)$ *is independent of* q.

(c) *If* $\frac{1}{\sqrt{xy}} < \alpha \le \underline{\alpha}(x,y)$, *then* $V_P^A(\alpha,q)$ *is increasing and concave in* q.

(d) *If* $\underline{\alpha}(x,y) < \alpha < \bar{\alpha}(x,y)$, *then* $V_P^A(\alpha,q)$ *is increasing and convex in* q *for* $q \in (0, \hat{q}(x,y))$, *and increasing and concave in* q *for* $q \in (\hat{q}(x,y), 1)$, *where* $\hat{q}(x,y) := \frac{\alpha - \underline{\alpha}(x,y)}{\bar{\alpha}(x,y) - \underline{\alpha}(x,y)}$.

(e) *If* $\bar{\alpha}(x,y) \le \alpha$, *then* $V_P^A(\alpha,q)$ *is increasing and convex in* q.

Proof. We treat the two cases −one in which both newcomer types are active and the other in which newcomer type x is inactive− separately.

Case 1: $\{\alpha > \eta(x,y)\} \cup \{\{\alpha < \eta(x,y)\} \cap \{q \in (\tilde{q}(x,y), 1]\}\}$. In this case, $V_P^A(\alpha,q) = 2\alpha\left(\frac{\mathbb{E}_q\left[\frac{1}{\sqrt{s}}\right]}{\alpha + \mathbb{E}_q\left[\frac{1}{s}\right]}\right)^2$. Consider the first- and the second-order derivatives of $V_P^A(\alpha,q)$ with respect to q:

$$\frac{dV_P^A(\alpha,q)}{dq} = \frac{4\alpha\left(\alpha - \frac{1}{\sqrt{xy}}\right)\left(\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{y}}\right)\mathbb{E}_q\left[\frac{1}{\sqrt{s}}\right]}{\left(\alpha + \mathbb{E}_q\left[\frac{1}{s}\right]\right)^3}, \tag{B.10}$$

$$\frac{d^2V_P^A(\alpha,q)}{dq^2} = \frac{4\alpha\left(\alpha - \frac{1}{\sqrt{xy}}\right)\left(\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{y}}\right)^2\left(\alpha - \left(\frac{3}{\sqrt{xy}} + 2\mathbb{E}_q\left[\frac{1}{s}\right]\right)\right)}{\left(\alpha + \mathbb{E}_q\left[\frac{1}{s}\right]\right)^4}. \tag{B.11}$$

From (B.10), it follows that $V_P^A(\alpha,q)$ is increasing, decreasing, and invariant with respect to q if α is greater than, less than, and equal to $\frac{1}{\sqrt{xy}}$, respectively.

Consider $\frac{1}{\sqrt{xy}} < \alpha$. Since $\frac{1}{y} \le \mathbb{E}_q\left[\frac{1}{s}\right] \le \frac{1}{x}$, we have $\underline{\alpha}(x,y) \le \frac{3}{\sqrt{xy}} + 2\mathbb{E}_q\left[\frac{1}{s}\right] \le \bar{\alpha}(x,y)$ and the equality holds on the left-hand-side (right-hand-side) if q equals zero (one). From (B.11), we find that $\frac{d^2V_P^A(\alpha,q)}{dq^2} < 0$ for all $q \in (0,1)$ if $\alpha \le \underline{\alpha}(x,y)$, $\frac{d^2V_P^A(\alpha,q)}{dq^2} > 0$ for all $q \in (0,1)$ if $\alpha \ge \bar{\alpha}(x,y)$, and $\frac{d^2V_P^A(\alpha,q)}{dq^2} \gtreqless 0$ for all $q \lesseqgtr \frac{\alpha - \frac{3}{\sqrt{xy}} - \frac{2}{y}}{2\left(\frac{1}{x} - \frac{1}{y}\right)} = \frac{\alpha - \underline{\alpha}(x,y)}{\bar{\alpha}(x,y) - \underline{\alpha}(x,y)} = \hat{q}(x,y)$, if $\underline{\alpha}(x,y) < \alpha < \bar{\alpha}(x,y)$.

Next, consider $\alpha < \frac{1}{\sqrt{xy}}$. From (B.11), we find that $\frac{d^2V_P^A(\alpha,q)}{dq^2} > 0$ for all $q \in (0,1)$, implying that $V_P^A(\alpha,q)$ is convex if $\alpha > \eta(x,y)$, or, if $\alpha < \eta(x,y)$ and $q \in (\tilde{q}(x,y), 1]$, i.e. whenever both newcomer types exert effort.

Case 2: $\{\alpha \le \eta(x,y)\} \cap \{q \in [0, \tilde{q}(x,y)]\}$. In this case, $V_P^A(\alpha,q) = 2\alpha y\left(\frac{1-p}{\alpha y + (1-p)}\right)^2$. Consider the first- and the second-order derivatives of $V_P^A(\alpha,q)$ with respect to q:

$$\frac{dV_P^A(\alpha,q)}{dq} = -\frac{4\alpha^2y^2(1-q)}{(\alpha y+(1-q))^3}, \tag{B.12}$$

$$\frac{d^2V_P^A(\alpha,q)}{dq^2} = \frac{4\alpha^2y^2(\alpha y-2(1-q))}{(\alpha y+(1-q))^4}. \tag{B.13}$$

From (B.12), it follows that $V_P^A(\alpha,q)$ is decreasing in q.

From (B.13) it is evident that $V_P^A(\alpha,q)$ is convex in q for $\eta(x,y)>\alpha>\frac{2}{y}$. Given that we have already proved convexity of $V_P^A(\alpha,q)$ for all $\eta(x,y)<\alpha<\frac{1}{\sqrt{xy}}$ in case 1, we can conclude that $V_P^A(\alpha,q)$ is convex in q for $min\left\{\frac{2}{y},\eta(x,y)\right\}<\alpha<\frac{1}{\sqrt{xy}}$.

Consider next $0<\alpha\leq min\left\{\frac{2}{y},\eta(x,y)\right\}$. In case 1, we have already established that in this range of values of α, $V_P^A(\alpha,q)$ is convex in q for $q\in(\tilde{q}(x,y),1]$, i.e., when both newcomer types exert effort. When the low ability newcomer is inactive, so that $V_P^A(\alpha,q)$ takes the form of $2\alpha y\left(\frac{1-p}{\alpha y+(1-p)}\right)^2$, it is concave in q for $\alpha y-2(1-q)<0$, or equivalently, for $q\in(0,q'(x,y))$, and convex otherwise, i.e., for $q\in(q'(x,y),1)$. Combining the two observations based on the form of $V_P^A(\alpha,q)$ together, we conclude that $V_P^A(\alpha,q)$ is concave in q for $q\in(0,min\{\tilde{q}(x,y),q'(x,y)\})$.

If $\tilde{q}(x,y)<q'(x,y)$, then $V_P^A(\alpha,q)=2\alpha y\left(\frac{1-p}{\alpha y+(1-p)}\right)^2$ for $q\in(0,\tilde{q}(x,y))$, and it is concave in q. For $q\in(\tilde{q}(x,y),1)$, then $V_P^A(\alpha,q)=2\alpha\left(\frac{\mathbb{E}_q\left[\frac{1}{\sqrt{s}}\right]}{\alpha+\mathbb{E}_q\left[\frac{1}{s}\right]}\right)^2$, which we have shown to be convex in q. If, on the other hand, $\tilde{q}(x,y)>q'(x,y)$, then $V_P^A(\alpha,q)=2\alpha y\left(\frac{1-p}{\alpha y+(1-p)}\right)^2$ for $q\in(0,\tilde{q}(x,y))$, and it is concave in q for $q\in(0,q'(x,y))$ and convex in q for $q\in(q'(x,y),\tilde{q}(x,y))$. As before, when $q\in(\tilde{q}(x,y),1)$, then $V_P^A(\alpha,q)=2\alpha\left(\frac{\mathbb{E}_q\left[\frac{1}{\sqrt{s}}\right]}{\alpha+\mathbb{E}_q\left[\frac{1}{s}\right]}\right)^2$, which is convex in q. Together, we can express $V_P^A(\alpha,q)$ as a piecewise convex function for $q\in(min\{\tilde{q}(x,y),q'(x,y)\},1)$ with two convex pieces defined over the two segments $(min\{\tilde{q}(x,y),q'(x,y)\},\tilde{q}(x,y))$ and $(\tilde{q}(x,y),1)$. □

As illustrated in the analysis of our basic model, for any given value of α, if $V_P^A(\alpha,q)$ is convex (concave) for all $q\in(0,1)$, the principal implements full (no) information disclosure. On the other hand, if $V_P^A(\alpha,q)$ changes its shape from convexity to concavity, or from concavity to convexity, or is piecewise convex, then the principal may be better off from partial information disclosure. Such a possibility arises if $0<\alpha\leq min\left\{\frac{2}{y},\eta(x,y)\right\}$ and if $\underline{\alpha}(x,y)<\alpha<\bar{\alpha}(x,y)$.

Consider $\underline{\alpha}(x,y)<\alpha<\bar{\alpha}(x,y)$.

In this case, $V_P^A(\alpha,q)$ is convex in q for $q\in(0,\hat{q}(x,y))$ and concave in q for $q\in$

$(\hat{q}(x,y),1)$. There exists a posterior $\mu_R(x,y,\alpha) \in [\hat{q}(x,y),1]$ such that $V_P^A(\alpha,q) < Cav(\alpha,q)$ for $q \in (0,\mu_R(x,y,\alpha))$ and $V_P^A(\alpha,q) = Cav(\alpha,q)$ for $q \in (\mu_R(x,y,\alpha),1)$. The posterior $\mu_R(x,y,\alpha)$ is given by

$$\mu_R(x,y,\alpha) := \begin{cases} p \in (0,1) : \frac{V_P^A(\alpha,p)-V_P^A(\alpha,0)}{p} = \frac{dV_P^A(\alpha,p)}{dp} & \text{if } V_P^A(\alpha,1) - V_P^A(\alpha,0) > \frac{dV_P^A(\alpha,1)}{dp} \\ 1 & \text{if } V_P^A(\alpha,1) - V_P^A(\alpha,0) \leq \frac{dV_P^A(\alpha,1)}{dp} \end{cases}. \tag{B.14}$$

The principal implements partial information disclosure for $q \in (0,\mu_R(x,y,\alpha))$ through generating two posteriors 0 and $\mu_R(x,y,\alpha)$, and no information disclosure for $q \in (\mu_R(x,y,\alpha),1)$. If $\mu_R(x,y,\alpha) = 1$, then full information disclosure is optimal. The signal distribution to generate the partial-information-disclosure posteriors is the same as the one we stated in our analysis of the basic model.

Consider $0 < \alpha \leq min\left\{\frac{2}{y},\eta(x,y)\right\}$.

In this case, $V_P^A(\alpha,q)$ is concave for $q \in (0,min\{\tilde{q}(x,y),q'(x,y)\})$, and piecewise convex for $q \in (min\{\tilde{q}(x,y),q'(x,y)\},1)$ over the two segments $(min\{\tilde{q}(x,y),q'(x,y)\},\tilde{q}(x,y))$ and $(\tilde{q}(x,y),1)$. Since the value of $V_P^A(\alpha,q)$ at $q = \tilde{q}(x,y)$ is always below the full disclosure payoff $V_P^F(\alpha,q)$, which follows from convexity of $V_P^A(\alpha,q)$ over $(\tilde{q}(x,y),1)$, the possibility of partial information disclosure arises only if $V_P^A(\alpha,q)$ is sufficiently concave in q for $q \in (0,min\{\tilde{q}(x,y),q'(x,y)\})$. In such a case, there exists a posterior $\mu_L(x,y,\alpha) \in [0,min\{\tilde{q}(x,y),q'(x,y)\}]$ such that $V_P^A(\alpha,q) = Cav(\alpha,q)$ for $q \in (0,\mu_L(x,y,\alpha))$ and $V_P^A(\alpha,q) < Cav(\alpha,q)$ for $q \in (\mu_L(x,y,\alpha),1)$. The posterior $\mu_L(x,y,\alpha)$ is given by

$$\mu_L(x,y,\alpha) := \begin{cases} p \in (0,1) : \frac{V_P^A(\alpha,p)-V_P^A(\alpha,1)}{p-1} = \frac{dV_P^A(\alpha,p)}{dp} & \text{if } -\frac{dV_P^A(\alpha,0)}{dp} < V_P^A(\alpha,0) - V_P^A(\alpha,1) \\ 0 & \text{if } -\frac{dV_P^A(\alpha,0)}{dp} \geq V_P^A(\alpha,0) - V_P^A(\alpha,1) \end{cases}. \tag{B.15}$$

The principal implements partial information disclosure for $q \in (\mu_L(x,y,\alpha),1)$ through generating two posteriors 1 and $\mu_L(x,y,\alpha)$, and no information disclosure for $q \in (0,\mu_L(x,y,\alpha))$. Full disclosure is optimal when $\mu_L(x,y,\alpha) = 0$; when this occurs is outlined in the following lemma.

Lemma B.3. *There exists an* $\hat{\alpha}(x,y) \in \left(0,\frac{1}{y}\right)$*, such that* $\mu_L(\alpha) = 0$ *for* $\alpha \in [\hat{\alpha}(x,y),\frac{1}{\sqrt{xy}})$*.*

Proof. Observe that $-\frac{dV_P^A(\alpha,0)}{dq} = \frac{4\alpha^2y^2}{(\alpha y+1)^3}$. Therefore,

$$-\frac{dV_P^A(\alpha,0)}{dp} \geq V_P^A(\alpha,0) - V_P^A(\alpha,1) \Leftrightarrow \frac{4\alpha^2 y^2}{(\alpha y+1)^3} \geq \frac{2\alpha y}{(\alpha y+1)^2} - \frac{2\alpha x}{(\alpha x+1)^2}$$
$$\Leftrightarrow 2\alpha y^2 \geq y(\alpha y+1) - \frac{x(\alpha y+1)^3}{(\alpha x+1)^2}$$
$$\Leftrightarrow \frac{x(\alpha y+1)^3}{y(\alpha x+1)^2} + \alpha y \geq 1$$

Define $f(\alpha) := \frac{x(\alpha y+1)^3}{y(\alpha x+1)^2} + \alpha y$. Then, $f'(\alpha) = \frac{x(\alpha y+1)^2}{y(\alpha x+1)^3}[\alpha xy + (3y-2x)] + \alpha > 0$ with $f(0) = \frac{x}{y} < 1$ and $f\left(\frac{1}{y}\right) = \frac{8xy}{(x+y)^2} + 1 > 1$. Hence, there exists an $\hat{\alpha}(x,y) \in \left(0, \frac{1}{y}\right)$, such that for $\alpha \in [\hat{\alpha}(x,y), \frac{1}{\sqrt{xy}})$, we have $-\frac{dV_P^A(\alpha,0)}{dp} \geq V_P^A(\alpha,0) - V_P^A(\alpha,1)$, or equivalently, $\mu_L(\alpha) = 0$. □

We now characterize the equilibrium information disclosure policy for a given degree of discrimination α. The proof follows from Lemma B.2 and the above discussion.

Proposition B.1. *Fix $\alpha > 0$, $0 < x < 1 < y$, and $q \in (0,1)$. The equilibrium information disclosure policy is characterized as follows:*

1. *Suppose $\alpha < \frac{1}{\sqrt{xy}}$.*
   (a) *If $\alpha \in (0, \hat{\alpha}(x,y))$, then the principal implements partial information disclosure for $q \in (\mu_L(x,y,\alpha), 1)$ through generating two posteriors 1 and $\mu_L(x,y,\alpha)$, and no information disclosure for $q \in (0, \mu_L(x,y,\alpha))$.*
   (b) *If $\alpha \in [\hat{\alpha}(x,y), \frac{1}{\sqrt{xy}})$, the principal implements full information disclosure.*
2. *Suppose $\alpha > \frac{1}{\sqrt{xy}}$.*
   (a) *If $\alpha \leq \underline{\alpha}(x,y)$, then the principal implements no information disclosure.*
   (b) *If $\underline{\alpha}(x,y) < \alpha < \bar{\alpha}(x,y)$, then the principal implements partial information disclosure for $q \in (0, \mu_R(x,y,\alpha))$ through generating two posteriors 0 and $\mu_R(x,y,\alpha)$, and no information disclosure for $q \in (\mu_R(x,y,\alpha), 1)$.*
   (c) *If $\bar{\alpha}(x,y) \leq \alpha$, then the principal implements full information disclosure.*
3. *Suppose $\alpha = \frac{1}{\sqrt{xy}}$. Then, the principal's expected payoff is invariant to any information disclosure policy.*

We let FD, ND denote the sets of values of α, for which the principal implements full-, and no-information disclosure in equilibrium, respectively. Partial discrimination can occur in two different regions for α, and these are denoted by PD_R for $\alpha > \frac{1}{\sqrt{xy}}$ and PD_L for $\alpha < \frac{1}{\sqrt{xy}}$.

B.4 Optimal discrimination

At stage 1, the principal's discrimination policy solves the following problem:

$$\max_{\alpha > 0} \; Cav\left(\alpha, q\right). \tag{B.16}$$

Denote the solution by $\alpha^*\left(x, y\right)$. We proceed in the following three steps.

Step 1: The principal's expected payoff from partial information disclosure is always dominated by her payoff from full- or no-information disclosure with a suitable choice of α.

Lemma B.4. *$\alpha^*\left(x,y\right) \notin PD_L \bigcup PD_R$. In particular, we have*

(a) *for $\alpha \in PD_R \bigcap \left(\underline{\alpha}\left(x,y\right), \infty\right)$, the principal's expected payoff from partial information disclosure is less than her expected payoff from no information disclosure with $\alpha = \underline{\alpha}\left(x,y\right)$.*

(b) *for $\alpha \in PD_L \bigcap \left(0, \hat{\alpha}\left(x,y\right)\right)$, the principal's expected payoff from partial information disclosure is less than her expected payoff from full information disclosure with $\alpha = \frac{1}{y}$.*

Proof. (a) First, we consider $\alpha \in PD_R \bigcap \left(\underline{\alpha}\left(x,y\right), \infty\right)$ and $q \in \left(0, \mu_R\left(x,y,\alpha\right)\right)$, and let $V_P^{PD_R}\left(\alpha, q\right)$ denote the principal's expected payoff under partial information disclosure. Further, by Proposition B.1, for all $\alpha \in PD_R$, $V_P^{PD_R}\left(\alpha, q\right) = Cav\left(\alpha, q\right)$. Since $Cav\left(\alpha, q\right)$ is the indirect value function of (B.9), we apply the envelope theorem to get

$$\frac{dCav\left(\alpha, q\right)}{d\alpha} = \sum_{m \in M} \widehat{\beta}_m \frac{\partial V_P^A\left(\alpha, \widehat{q}_m\right)}{\partial \alpha},$$

where $\widehat{\beta}_m \in [0,1]$ and $\widehat{q}_m \in [0,1]$ are solutions of (B.9). Note that for all $\alpha \in PD_R$, these solutions are given by: $\widehat{\beta}_1 = \frac{q}{\mu_R(x,y,\alpha)}, \widehat{\beta}_2 = 1 - \frac{q}{\mu_R(x,y,\alpha)}; \widehat{q}_1 = \mu_R\left(x,y,\alpha\right), \widehat{q}_2 = 0$. Therefore,

$$\frac{dV_P^{PD_R}\left(\alpha, q\right)}{d\alpha} = \frac{q}{\mu_R\left(x,y,\alpha\right)} \frac{\partial V_P^A\left(\alpha, \mu_R\left(\alpha\right)\right)}{\partial \alpha} + \left(1 - \frac{q}{\mu_R\left(x,y,\alpha\right)}\right) \frac{\partial V_P^A\left(\alpha, 0\right)}{\partial \alpha}. \tag{B.17}$$

We show that both $\frac{\partial V_P^A(\alpha, \mu_R(\alpha))}{\partial \alpha}$ and $\frac{\partial V_P^A(\alpha, 0)}{\partial \alpha}$ are negative for $\alpha \in PD_R \bigcap \left(\underline{\alpha}\left(x,y\right), \infty\right)$. Note that $\frac{\partial V_P^A(\alpha, q)}{\partial \alpha} = \frac{2\mathbb{E}_q^2\left[\frac{1}{\sqrt{s}}\right]\left[\mathbb{E}_q\left[\frac{1}{s}\right] - \alpha\right]}{\left(\alpha + \mathbb{E}_q\left[\frac{1}{s}\right]\right)^3}$. Therefore, $\frac{\partial V_P^A(\alpha, 0)}{\partial \alpha} = \frac{2\mathbb{E}_{p=0}^2\left[\frac{1}{\sqrt{s}}\right]\left[\frac{1}{y} - \alpha\right]}{\left(\alpha + \mathbb{E}_{p=0}\left[\frac{1}{s}\right]\right)^3} < 0$, since $\frac{1}{y} <$

$\underline{\alpha}(x,y) < \alpha$. Further, $\frac{\partial V_P^A(\alpha,\mu_R(x,y,\alpha))}{\partial\alpha} = \frac{2\mathbb{E}^2_{\mu_R(x,y,\alpha)}\left[\frac{1}{\sqrt{s}}\right]\left[\mathbb{E}_{\mu_R(x,y,\alpha)}\left[\frac{1}{s}\right]-\alpha\right]}{\left(\alpha+\mathbb{E}_{\mu_R(x,y,\alpha)}\left[\frac{1}{s}\right]\right)^3}$, which is negative if and only if $\left[\mathbb{E}_{\mu_R(x,y,\alpha)}\left[\frac{1}{s}\right]-\alpha\right]$ is negative. From (B.14), direct calculation gives

$$\mu_R(x,y,\alpha) \quad = \quad \frac{yx\sqrt{x}\,(1+\alpha y)\,(\alpha-\underline{\alpha}(x,y))}{(y-x)\left(2\sqrt{x}+\sqrt{y}+\alpha y\sqrt{x}\right)},$$

and

$$\mathbb{E}_{\mu_R(x,y,\alpha)}\left[\frac{1}{s}\right]-\alpha = \frac{1}{y}+\mu_R(x,y,\alpha)\left(\frac{1}{x}-\frac{1}{y}\right)-\alpha = -\frac{4\alpha y+2\alpha\sqrt{xy}+2}{\sqrt{y}\left(2\sqrt{x}+\sqrt{y}+\alpha y\sqrt{x}\right)},$$

which is negative. Hence, $\frac{dV_P^{PD}(\alpha,q)}{d\alpha}$ is negative for $\alpha \in PD_R \bigcap (\underline{\alpha}(x,y),\infty)$. Further, since at $\alpha=\underline{\alpha}(x,y)$, $Cav(\alpha,q)=V_P^A(\alpha,q)$ and it is continuous, we conclude that the principal's expected payoff from partial information disclosure is less than her expected payoff from no information disclosure with $\alpha=\underline{\alpha}(x,y)$.

(b) Next, consider $\alpha \in PD_L \bigcap (0,\hat{\alpha}(x,y))$ and $q \in (\mu_L(x,y,\alpha),1)$. We show that any choice of $\alpha \in PD_L$ gives the principal a lower expected payoff than choosing $\alpha=\frac{1}{y}$ and full information disclosure, which yields a payoff of $V_P^F\left(\frac{1}{y},q\right)$. Then $V_P^F\left(\frac{1}{y},1\right) = \frac{2xy}{(y+x)^2} > \frac{2\alpha x}{(\alpha x+1)^2} = V_P^A(\alpha,1)$ for $\alpha < \hat{\alpha}(x,y) < \frac{1}{y}$. Further, by definition, $\mu_L(x,y,\alpha) < \tilde{q}(x,y)$ and for $q < \tilde{q}(x,y)$, $V_P^A(\alpha,q) = 2\alpha y\left(\frac{1-q}{\alpha y+(1-q)}\right)^2$ which is a concave function in α, with a maximum at $\alpha = \frac{1-q}{y}$. The maximum value of this function, measured at $q=\mu_L(x,y,\alpha)$ is $V_P^A\left(\frac{1-q}{y},\mu_L(x,y,\alpha)\right) = \frac{1-\mu_L(x,y,\alpha)}{2}$. Furthermore, $V_P^F\left(\frac{1}{y},\mu_L(x,y,\alpha)\right) - V_P^A\left(\frac{1-q}{y},\mu_L(x,y,\alpha)\right) = \frac{2xy\mu_L(x,y,\alpha)}{(y+x)^2} > 0$. Since $V_P^F\left(\frac{1}{y},q\right) = qV_P^A\left(\frac{1}{y},1\right)+(1-q)\,V_P^A\left(\frac{1}{y},0\right)$ and $\frac{1-q}{1-\mu_L(x,y,\alpha)}V_P^A(\alpha,\mu_L(x,y,\alpha))+\left(1-\frac{1-q}{1-\mu_L(x,y,\alpha)}\right)V_P^A(\alpha,1)$ both represent straight lines in (V,q) space, and two points on the former are always strictly above the latter, it follows that setting $\alpha=\frac{1}{y}$ with full disclosure can always yield a larger expected payoff than selecting $\alpha \in PD_L \bigcap (0,\hat{\alpha}(x,y))$ with partial disclosure.

Together, we conclude that the optimal choice of discrimination parameter will not be associated with partial information disclosure policy, i.e., $\alpha^*(x,y) \notin PD_L \bigcup PD_R$. □

Step 2: We show that the optimal discrimination parameter lies in the interval $\left[\frac{1}{y},\frac{1}{x}\right]$.

Lemma B.5. *For any $q \in (0,1)$, $\alpha^*(x,y) \in \left[\frac{1}{y},\frac{1}{x}\right]$.*

Proof. The proof is done in two steps. First, consider that $\alpha > \frac{1}{\sqrt{xy}}$. Since $Cav(\alpha,q)$ is the indirect value function of (B.9), applying the envelope theorem, we get that

$$\frac{dCav\left(\alpha,q\right)}{d\alpha}=\sum_{m\in M}\widehat{\beta}_m\frac{\partial V_P^A\left(\alpha,\widehat{q}_m\right)}{\partial\alpha},\tag{B.18}$$

where $\widehat{\beta}_m\in[0,1]$ and $\widehat{q}_m\in[0,1]$ are solutions of (B.9). Further, for any arbitrary distribution $p\in[0,1]$,

$$\frac{\partial V_P^A\left(\alpha,p\right)}{\partial\alpha}=\frac{\partial}{\partial\alpha}2\alpha\left(\frac{\mathbb{E}_p\left[\frac{1}{\sqrt{s}}\right]}{\alpha+\mathbb{E}_p\left[\frac{1}{s}\right]}\right)^2=2\mathbb{E}_p^2\left[\frac{1}{\sqrt{s}}\right]\frac{\partial}{\partial\alpha}\left[\frac{\alpha}{\left(\alpha+\mathbb{E}_p\left[\frac{1}{s}\right]\right)^2}\right]=\frac{2\mathbb{E}_p^2\left[\frac{1}{\sqrt{s}}\right]\left[\mathbb{E}_p\left[\frac{1}{s}\right]-\alpha\right]}{\left(\alpha+\mathbb{E}_p\left[\frac{1}{s}\right]\right)^3},\tag{B.19}$$

which is strictly positive for $\alpha<\mathbb{E}_p\left[\frac{1}{s}\right]$ and strictly negative for $\alpha>\mathbb{E}_p\left[\frac{1}{s}\right]$. Since, for any $\widehat{q}_m\in[0,1]$, $\mathbb{E}_{\widehat{q}_m}\left[\frac{1}{s}\right]\in\left[\frac{1}{y},\frac{1}{x}\right]$, (B.18) is strictly positive for $\alpha<\frac{1}{y}$ and strictly negative for $\alpha>\frac{1}{x}$. Hence, if $\alpha^*\left(x,y\right)>\frac{1}{\sqrt{xy}}$, it must lie in $\left(\frac{1}{\sqrt{xy}},\frac{1}{x}\right]$.

Next, consider that $\alpha<\frac{1}{\sqrt{xy}}$. Lemma (B.4) has established that in this region, $\alpha^*\left(x,y\right)$ cannot be associated with partial-information disclosure. We now show that it cannot be associated with a no-information disclosure policy as well. From Proposition B.1, we know that $Cav\left(\alpha,q\right)=V_P^A\left(\alpha,q\right)=2\alpha x\left(\frac{1-q}{\alpha+x(1-q)}\right)^2$ for $\alpha\in ND\bigcap\left(0,\hat{\alpha}\left(x,y\right)\right)$ and $q\in\left(0,\mu_L\left(x,y,\alpha\right)\right)$. Lemma B.4 shows that $V_P^A\left(\alpha,q\right)$ is maximized at $\alpha=\frac{1-q}{y}$, with maximum value $V_P^A\left(\frac{1-q}{y},q\right)=\frac{1-q}{2}$. And, by setting $\alpha=\frac{1}{y}$ and following a full disclosure policy, the principal receives an expected payoff of $V_P^F\left(\frac{1}{y},q\right)=qV_P^A\left(\frac{1}{y},1\right)+\left(1-q\right)V_P^A\left(\frac{1}{y},0\right)=qV_P^A\left(\frac{1}{y},1\right)+\frac{1-q}{2}$, since $V_P^A\left(\frac{1}{y},0\right)=\frac{1}{2}$. Recall that $V_P^A\left(\frac{1}{y},1\right)$ is given by $\frac{2}{y}\left(\frac{\mathbb{E}_{p=1}\left[\frac{1}{\sqrt{s}}\right]}{\frac{1}{y}+\mathbb{E}_{p=1}\left[\frac{1}{s}\right]}\right)^2>0$. Hence $V_P^F\left(\frac{1}{y},q\right)>\frac{1-q}{2}$, and full disclosure with $\alpha=\frac{1}{y}$ gives the principal a higher expected payoff than setting $\alpha\in ND\cap\left(0,\hat{\alpha}\left(x,y\right)\right)]$ with no-information disclosure.

Therefore, for $\alpha<\frac{1}{\sqrt{xy}}$, the optimal choice of discrimination parameter must be associated with full information disclosure and hence the expected payoff of the principal is $V_P^F\left(\alpha,p\right)$. For any arbitrary distribution $p\in[0,1]$,

$$\frac{\partial V_P^F\left(\alpha,p\right)}{\partial\alpha}=2\mathbb{E}_p\left[\frac{s\left(1-\alpha s\right)}{\left(1+\alpha x\right)^3}\right],$$

which is strictly positive for $\alpha\in\left[\frac{1}{y},\frac{1}{x}\right]$. Hence, if $\alpha^*\left(x,y\right)<\frac{1}{\sqrt{xy}}$, it must lie in $\left[\frac{1}{y},\frac{1}{\sqrt{xy}}\right)$. Since $\frac{1}{y}<\frac{1}{\sqrt{xy}}<\frac{1}{x}$, the above observations together imply that $\alpha^*\left(x,y\right)\in\left[\frac{1}{y},\frac{1}{x}\right]$. □

Step 3: As a direct implication of Proposition B.1 and above lemmas, we show that the principal follows full-information (no-information) disclosure policy if her choice of discrimination parameter α is below (above) $\frac{1}{\sqrt{xy}}$.

Corollary B.1. *If* $\frac{1}{y} \leq \alpha^*(x,y) < \frac{1}{\sqrt{xy}}$*, then the principal implements full information disclosure; if* $\frac{1}{\sqrt{xy}} \leq \alpha^*(x,y) < \frac{1}{x}$*, then the principal implements no information disclosure.*

Proof. Proposition B.1, Lemma B.4, and analysis of the case $\alpha < \frac{1}{\sqrt{xy}}$ in the proof of Lemma B.5 together imply that if $\alpha^*(x,y) < \frac{1}{\sqrt{xy}}$, $\alpha^*(x,y)$ must be associated with a full-information disclosure policy and if $\alpha^*(x,y) > \frac{1}{\sqrt{xy}}$, $\alpha^*(x,y)$ must be associated with a no-information disclosure policy. □

Following the three steps, $\alpha^*(x,y)$ can be easily determined by comparing the principal's maximum payoff from full disclosure for $\alpha \in \left[\frac{1}{y}, \frac{1}{\sqrt{xy}}\right)$ and the principal's maximum payoff from no disclosure for $\alpha \in \left(\frac{1}{\sqrt{xy}}, \frac{1}{x}\right]$. We denote the principal's optimal choice of discrimination under full- and no-information disclosure policy by $\alpha^{FD}(x,y)$ and $\alpha^{ND}(x,y)$. Formally,

Definition B.1. $\alpha^{FD}(x,y) := \underset{\alpha>0}{\arg\max}\ V_P^F(\alpha,q)$ *and* $\alpha^{ND}(x,y) := \underset{\alpha\geq\frac{1}{\sqrt{xy}}}{\arg\max}\ V_P^A(\alpha,q)$.

There will be two local maxima of $V_P^A(\alpha,q)$, depending on whether or not both newcomer types are active. Since we have already established that $\alpha^*(x,y)$ is associated with a no-disclosure policy only when $\alpha > \frac{1}{\sqrt{xy}}$, which rules out the possibility of implementing no disclosure with only one newcomer type being active, we define α^{ND} as the argument of the maxima over a constrained set of $\alpha \geq \frac{1}{\sqrt{xy}}$. We show that $\alpha^{ND}(x,y)$ is then uniquely defined. The following lemmas characterize properties of $\alpha^{FD}(x,y)$ and $\alpha^{ND}(x,y)$.

Lemma B.6. α^{FD} *solves* $\mathbb{E}_q\left[\frac{s(1-\alpha s)}{(1+\alpha s)^3}\right]=0$*. Further,* $\alpha^{FD} \lesseqqgtr \frac{1}{\sqrt{xy}} \Leftrightarrow q \lesseqqgtr \frac{1}{2}$.

Proof. The following two claims are useful in proving the result.

Claim 1. $V_P^F\left(\frac{\alpha}{\sqrt{xy}}, q\right) = V_P^F\left(\frac{1}{\alpha\sqrt{xy}}, 1-q\right)$.

Proof of Claim 1:

$$
\begin{aligned}
&V_P^F\left(\frac{1}{\alpha\sqrt{xy}}, 1-q\right)\\
&=\mathbb{E}_{1-q}\left[\frac{2\frac{s}{\alpha\sqrt{xy}}}{\left(1+\frac{s}{\alpha\sqrt{xy}}\right)^2}\right] = \mathbb{E}_{1-q}\left[\frac{2\frac{\alpha\sqrt{xy}}{s}}{\left(\frac{\alpha\sqrt{xy}}{s}+1\right)^2}\right]\\
&=(1-q)\left[\frac{2\alpha\sqrt{\frac{y}{x}}}{\left(\alpha\sqrt{\frac{y}{x}}+1\right)^2}\right] + q\left[\frac{2\alpha\sqrt{\frac{x}{y}}}{\left(\alpha\sqrt{\frac{x}{y}}+1\right)^2}\right] = \mathbb{E}_q\left[\frac{2\frac{\alpha}{\sqrt{xy}}s}{\left(\frac{\alpha}{\sqrt{xy}}s+1\right)^2}\right] = V_P^F\left(\frac{\alpha}{\sqrt{xy}}, q\right).
\end{aligned}
$$

Claim 2. $V_P^F(\alpha, q) \gtreqless V_P^F(\alpha, 1-q) \Leftrightarrow \left(q - \frac{1}{2}\right)\left(\alpha - \frac{1}{\sqrt{xy}}\right) \gtreqless 0.$

Proof of Claim 2:

$$\begin{aligned}
&V_P^F(\alpha, q) - V_P^F(\alpha, 1-q) \\
&= q\left[\frac{2\alpha x}{(\alpha x+1)^2}\right] + (1-q)\left[\frac{2\alpha y}{(\alpha y+1)^2}\right] - (1-q)\left[\frac{2\alpha x}{(\alpha x+1)^2}\right] - q\left[\frac{2\alpha y}{(\alpha y+1)^2}\right] \\
&= (1-2q)\left[\frac{2\alpha y}{(\alpha y+1)^2} - \frac{2\alpha x}{(\alpha x+1)^2}\right] = \frac{2\alpha(1-2q)}{(\alpha y+1)^2(\alpha x+1)^2}\left[y(\alpha x+1)^2 - x(\alpha y+1)^2\right] \\
&= \frac{2\alpha(y-x)(2q-1)(xy\alpha^2-1)}{(\alpha y+1)^2(\alpha x+1)^2} = \frac{4\alpha xy(y-x)\left(q-\frac{1}{2}\right)\left(\alpha+\frac{1}{\sqrt{xy}}\right)\left(\alpha-\frac{1}{\sqrt{xy}}\right)}{(\alpha y+1)^2(\alpha x+1)^2}.
\end{aligned}$$

Since $x < y$, we get $V_P^F(\alpha, q) \gtreqless V_P^F(\alpha, 1-q) \Leftrightarrow \left(q - \frac{1}{2}\right)\left(\alpha - \frac{1}{\sqrt{xy}}\right) \gtreqless 0$.This completes the proof of Claim 2.

From Claim 2, replacing α by $\frac{1}{\alpha\sqrt{xy}}$ and q by $1-q$, we get

$$\begin{aligned}
V_P^F\left(\frac{1}{\alpha\sqrt{xy}}, 1-q\right) \gtreqless V_P^F\left(\frac{1}{\alpha\sqrt{xy}}, q\right) &\Leftrightarrow \left((1-q) - \frac{1}{2}\right)\left(\frac{1}{\alpha\sqrt{xy}} - \frac{1}{\sqrt{xy}}\right) \gtreqless 0 \\
&\Leftrightarrow \left(\frac{1}{2} - q\right)(1-\alpha) \gtreqless 0.
\end{aligned}$$

By Claim 1, we replace $V_P^F\left(\frac{1}{\alpha\sqrt{xy}}, 1-q\right)$ by $V_P^F\left(\frac{\alpha}{\sqrt{xy}}, q\right)$ to get

$$V_P^F\left(\frac{\alpha}{\sqrt{xy}}, q\right) \gtreqless V_P^F\left(\frac{1}{\alpha\sqrt{xy}}, q\right) \Leftrightarrow \left(\frac{1}{2} - q\right)(1-\alpha) \gtreqless 0.$$

Finally, we replace α by $\alpha^{FD}\sqrt{xy}$ to get

$$V_P^F\left(\alpha^{FD}, q\right) \gtreqless V_P^F\left(\frac{1}{\alpha^{FD}xy}, q\right) \Leftrightarrow \left(\frac{1}{2} - q\right)\left(1 - \alpha^{FD}\sqrt{xy}\right) \gtreqless 0. \tag{B.20}$$

By definition of α^{FD}, $V_P^F\left(\alpha^{FD}, q\right) \geq V_P^F(\alpha, q)$ for any α, and in particular, for $\alpha = \frac{1}{\alpha^{FD}xy}$. Therefore, (B.20) implies $\left(\frac{1}{2} - q\right)\left(1 - \alpha^{FD}\sqrt{xy}\right) \geq 0$, and equivalently, $\alpha^{FD} \lesseqgtr \frac{1}{\sqrt{xy}} \Leftrightarrow q \lesseqgtr \frac{1}{2}$. □

Lemma B.7. $\alpha^{ND} = \begin{cases} \mathbb{E}_q\left[\frac{1}{s}\right] & \textit{if } q > \frac{\sqrt{x}}{\sqrt{y}+\sqrt{x}} \\ \frac{1}{\sqrt{xy}} & \textit{if } q \leq \frac{\sqrt{x}}{\sqrt{y}+\sqrt{x}} \end{cases}.$

Proof. Note that for $\alpha \geq \frac{1}{\sqrt{xy}}$, both newcomer types are active and $V_P^A(\alpha, q) = 2\alpha\left(\frac{\mathbb{E}_p\left[\frac{1}{\sqrt{s}}\right]}{\alpha + \mathbb{E}_p\left[\frac{1}{s}\right]}\right)^2$.

The derivative of $V_P^A(\alpha, q)$ with respect to α:

$$\frac{dV_P^A(\alpha,q)}{d\alpha} = 2\mathbb{E}_q^2\left[\frac{1}{\sqrt{s}}\right]\frac{d}{d\alpha}\left[\frac{\alpha}{\left(\alpha+\mathbb{E}_q\left[\frac{1}{s}\right]\right)^2}\right] = \frac{2\mathbb{E}_q^2\left[\frac{1}{\sqrt{s}}\right]\left[\mathbb{E}_q\left[\frac{1}{s}\right]-\alpha\right]}{\left(\alpha+\mathbb{E}_q\left[\frac{1}{s}\right]\right)^3}. \tag{B.21}$$

Setting the derivative to zero, we get a local optimum of the unconstrained problem at $\alpha = \mathbb{E}_q\left[\frac{1}{s}\right]$. Further, it follows from (B.21) that $V_P^A(\alpha, q)$ is increasing for $\alpha < \mathbb{E}_q\left[\frac{1}{s}\right]$ and decreasing for $\alpha > \mathbb{E}_q\left[\frac{1}{s}\right]$, implying that $\mathbb{E}_q\left[\frac{1}{s}\right]$ is a global maximum of the unconstrained problem. If $\mathbb{E}_q\left[\frac{1}{s}\right] > \frac{1}{\sqrt{xy}}$, the argument of the constrained maxima α^{ND} coincides with $\mathbb{E}_q\left[\frac{1}{s}\right]$, and if $\mathbb{E}_q\left[\frac{1}{s}\right] \leq \frac{1}{\sqrt{xy}}$, α^{ND} is given by $\frac{1}{\sqrt{xy}}$. Finally, the statement of the lemma follows from the fact that $\mathbb{E}_q\left[\frac{1}{s}\right] \lesseqqgtr \frac{1}{\sqrt{xy}} \Leftrightarrow q \lesseqqgtr \frac{\sqrt{x}}{\sqrt{y}+\sqrt{x}}$. □

We now state the main proposition, which describes the equilibrium discrimination policy.

Proposition B.2. *Fix $0 < x < 1 < y$ and $q \in (0, 1)$. There exists a threshold $\overline{q}(x, y) \in \left[\frac{\sqrt{x}}{\sqrt{y}+\sqrt{x}}, \frac{1}{2}\right]$ such that*

1. *If $q < \overline{q}(x, y)$, then the principal's choice of discrimination is $\alpha^*(x, y) = \alpha^{FD}(x, y) < \frac{1}{\sqrt{xy}}$ and there is full information disclosure in equilibrium.*
2. *If $q > \overline{q}(x, y)$, then the principal's choice of discrimination is $\alpha^*(x, y) = \alpha^{ND}(x, y) > \frac{1}{\sqrt{xy}}$ and there is no information disclosure in equilibrium.*
3. *If $q = \overline{q}(x, y)$, then the principal is indifferent between choosing $\alpha(x, y) = \alpha^{FD}(x, y) < \frac{1}{\sqrt{xy}}$ along with full information disclosure and choosing $\alpha = \alpha^{ND}(x, y) > \frac{1}{\sqrt{xy}}$ along with no information disclosure.*

Proof. This proof is identical to the corresponding proposition in the main paper, with the following modifications: The lower limit of $\overline{q}(x, y)$ is now given by $\frac{\sqrt{x}}{\sqrt{y}+\sqrt{x}}$, since at $q = \frac{\sqrt{x}}{\sqrt{y}+\sqrt{x}}$, we have $\mathbb{E}_q\left[\frac{1}{s}\right] = \frac{1}{\sqrt{xy}}$. □